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# SUPER FELICITOUS DIFFERENCE LABELING OF SPECIAL TYPES OFGRAPHS

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**Abstract**

A graph with p vertices and q edges is called super felicitous difference labeling graph if f: V (G) $\rightarrow \{1,2, \ldots \ldots p+q\}$is an injective map so that the induced edge labeling is defined by$f^*(e=uv) = \big(f(u)-f(v)\big) \pmod{p+q}$and$f\big(v(G)\big) \cup f^*(e): e \in E(G) = \{1,2,3 \ldots .p+q\}$. A graph that admits a **SuperFelicitous Difference Labeling**(SFDL) is called **Super Felicitous Difference Labeling Graph**. In this paper, we investigate super felicitous difference labeling graph of special types of trees like the Jelly fish, thespider graph, the Globe graph Gl(n),the ladder graph,the $H_{n,n}$graph, $S'(P_n)$, the $P_2(+)N_{2n}$ Graph and the Jewel graph.

**Keywords:**SuperFelicitous Difference Labeling (SFDL), Felicitous Difference labeling graph

## I.INTRODUCTION

All graphs in this paper represent finite, undirected and simple one. The vertex set and the edge set of a graph G are denoted by V(G) and E(G) respectively. Terms and Notations not defined here are used in the sense of Harary[5].

A graph labeling is an assignment of integers to the vertices or edges or both vertices and edges subject to certain conditions. There are several types of graph labeling and a detailed survey is found in [6]. The notion of felicitous difference labeling was due to V. Lakshmi Alias Gomathi, A. Nagarajan and A. NellaiMurugan[8].

In this paper, we define super felicitous difference labeling graph and show that the Jelly fish, the spider graph, the Globe graphGl(n), the ladder graph,the $H_{n,n}$ graph, $S'(P_n)$graph, the $P_2(+)N_{2n}$and the Jewel Graph are super felicitous difference labeling graph. We use the following definitions in the subsequent section.

## II.Preliminaries

**Definition 2.1:** The Jelly fish graph (m,n) is obtained from a 4 – cycle $(v_1, v_2, v_3, v_4)$ together with an edge $v_1, v_3$ and appending m pendent edges to $v_2$ and n pendent edges to $v_4$.

**Definition 2.2:**A Spideris a tree having a unique node e with degree greater than 2 and all the other nodes have degrees less than or equal to 2.

A Spiderwith k legs of length $n_i$: $1 \leq i \leq k$ is denoted by SP ( $n_1, n_2, \ldots n_k$). The graph SP ( $P_n$, m)denotes a spider having a path $P_n$ with m pendant vertices attached to one end vertex of $P_n$. A spider with k legs (paths) each of length n is called a regular spider and is denoted by SP (k, n).

**Definition 2.3:**A globe is a graph obtained from two isolated vertex are joined by n paths of length two. It is denoted by Gl(n).

**Definition 2.4:**The ladder $L_n$ (n≥ 2) is the product graph $P_2 \times P_n$which contains 2n vertices and 3n – 2 edges.

**Definition 2.5:** The graph $H_{n,n}$ is a special bipartite graph with the vertex set

V $(H_{n,n}) = \{u_i, v_i: 1 \leq i \leq n\}$ and edge set E$(H_{n,n}) = \{(u_i, v_i): 1 \leq i \leq n \; and \; n - i + 1 \leq j \leq n\}$.

**Definition 2.6:**For a graph G, the Splitting graph which is denoted by Spl(G) is obtained by adding to each vertex u, a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G.

**Definition 2.7:** A Path $P_n$ is a walk in which all the vertices are distinct.

**Definition 2.8:**The Jewel graph$J_n$ is the graph with vertex set $V(J_n) = \{u, v, x, y, u_i / 1 \leq i \leq n\}$ and edge set $E(J_n) = \{ux, uy, xv, yv, uu_i, vv_i / 1 \leq i \leq n\}$.

**III. Main Results**

**Definition 3.1:** A graph with p vertices and q edges is called super felicitous difference labeling graph if f: V (G) → $\{1,2, \ldots . p + q\}$is an injective map so that the induced edge labeling is defined byf$^*(e = uv) = (f(u) - f(v)) \pmod{p + q}$andf$(v(G)) \cup f^*(e): e \in E(G) = \{1,2,3 \ldots . p + q\}$. A graph that admits a **SuperFelicitous Difference Labeling**(SFDL) is called **Super Felicitous Difference Labeling Graph**.

**Theorem 3.2:** The Jelly fish J(m,2n) is SFDL graph for all m and n .

**Proof:** Let G be the graph J(m,2n).

Let V (G)= $\left\{ u, v, x, y, u_i, v_j \middle/ 1 \leq i \leq m, 1 \leq j \leq 2n \right\}$ and

E (G) =$\{xu, xv, yu, yv, xy\} \cup \left\{ uu_i \middle/ 1 \leq i \leq m \right\} \cup \left\{ vv_j \middle/ 1 \leq j \leq 2n \right\}$Then |V $(G)$| = m+n+4 and |E $(P_n)$| = m+n+5.

Define f: V (G)→ $\{1,2, \ldots . 3(m + n + 1) + 3, 3(m + n + 1) + 6\}$ as follows:

$f(u) = 1$

$f(v) = 2$

$f(x) = 3(m + n + 1) + 3$

$f(y) = 3(m + n + 1) + 6$

$f(u_i) = \{2k + 1, 2 \leq k \leq m + 1$

$f(v_1) = f(u_m) + 3$

$f(v_{2k+1}) = f(v_{2k+1-2}) + 4, \quad 1 \leq k \leq m - 1$

$f(v_2) = f(u_m) + 4$

$f(v_{2k}) = f(v_{2k-2}) + 4, \quad 2 \leq k \leq m - 1$

let $f^*$ be the induced edge labeling of f. Then

$f^*(uu_i) = 2k \qquad 2 \le k \le m+1$
$f^*(vv_j) = f(v_j) - f(v) 1 \le j \le 2n$
$f^*(xu) = 3(m+n+1) + 2$
$f^*(xv) = 3(m+n+1) + 1$
$f^*(yu) = 3(m+n+1) + 5$
$f^*(yv) = 3(m+n+1) + 4$
$f^*(xy) = 3$

Then the induced edge labels are all distinct. Hence from the above labeling pattern, the Jelly fish graph J(m,2n) admits Super felicitous difference labeling graph.

**Example 3.2.1:** The SFD labeling graph of J(3,6) is given in fig.1

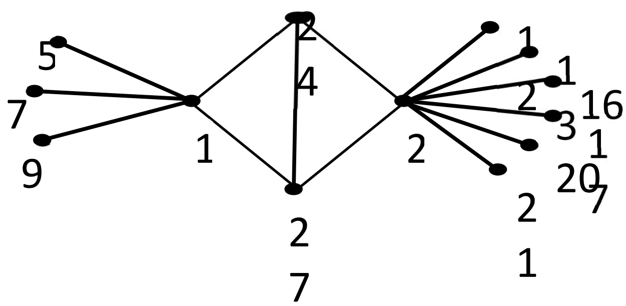



**Fig. 1**

**Theorem 3.3:** Spider SP $(P_m, k_{1,n})$ is a SFDL graph for all m,n.

**Proof:** Let V (SP $(P_m, k_{1,n})$) $= \left\{ {u_i}/{1 \le i \le m} \right\} \cup \left\{ {v_j}/{1 \le j \le n} \right\}$ and

E (SP $(P_m, k_{1,n})$) $= \left\{ {u_i u_{i+1}}/{1 \le i \le m-1} \right\} \cup \left\{ {v_0 v_j}/{1 \le j \le n} \right\}$

Let $u_m = v_0$be the centre vertex of $k_{1,n}$.

Define f: V (SP $(P_m, k_{1,n})$) → $\{1, 2, \ldots..2m+2n-3, 2m+2n-1\}$ as follows:

$f(v_j) = 2m + 2n - 1 - 2j \qquad 0 \le j \le m-1$

**Case (i):** when m is odd

$f(u_{2i+1}) = m - 2i \qquad 0 \le i \le \frac{m+1}{2}$

$f(u_{2i}) = m + 2i \qquad 0 \le i \le \frac{m-1}{2}$

**Case (ii):** when m is even

$f(u_{2i+1}) = (m+1) + 2i \qquad 0 \le i \le \frac{m}{2}$

$f(u_{2i}) = (m-1) - 2i \quad 0 \le i \le \frac{m}{2}$

Then the induced edge labels are distinct. Hence from the above labeling pattern the graph SP $(P_m, k_{1,n})$ admits Super Felicitous Difference labeling graph.

**Example 3.3.1:** The SFD labeling graph of SP $(P_8, k_{1,5})$ is given in fig. 2.

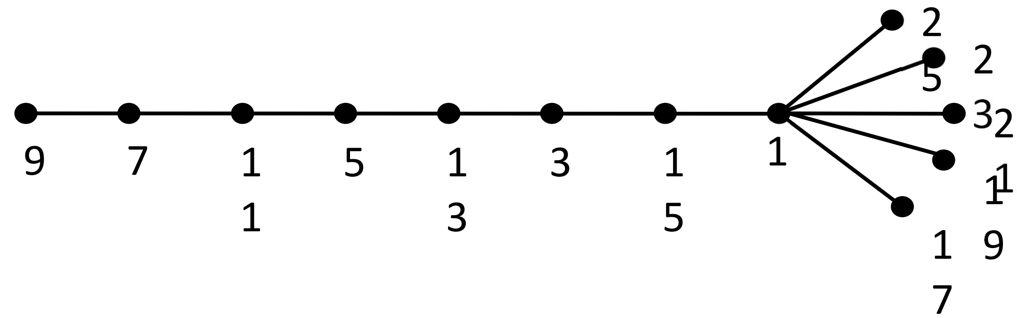



**Fig. 2**

**Theorem 3.4:** Globe Gl(n)is SFDL graph for all m and n.

**Proof:** Let V (Gl(n))$\{u, v, w_i: 1 \leq i \leq n\}$ and

E (Gl(n)) =$\left\{{uw_i}/_{1 \leq i \leq n}\right\} \cup \left\{{vw_i}/_{1 \leq i \leq 2n}\right\}$

Define f: V (Gl(n)) → $\{1,2, \ldots\ldots 3n + 2\}$ as follows:

f(u) = 1

f(v) = 2

f$(w_i) = 3n + 2 - 3h,\ \ 0 \leq h \leq \frac{2n-2}{2}$

let $f^*$be the induced edge labels

$$f^*(uw_i) = 3n + 1 - 3h, \quad 0 \leq h \leq \frac{2n-2}{2}$$

$$f^*(vw_i) = 3n - 3h, \quad\quad 0 \leq h \leq \frac{2n-2}{2}$$

Then the induced edge labels are distinct. Hence from the above labeling pattern, the graph Gl(n)admits super felicitous difference labeling graph.

**Example 3.4.1:** The SFD labeling graph of Gl(6) isgiven in fig 3 respectively.

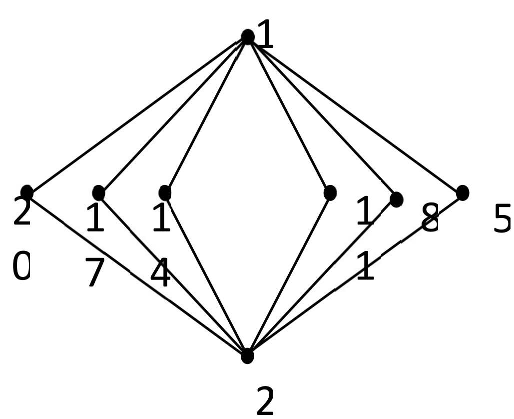



**Fig. 3**

**Theorem 3.5:** Ladders n ≤ 5 are Super Felicitous Difference Labeling Graph.

**Proof:**Assume $n \leq 3$

**Case (i):**

let $V(L_n) = \{u_i v_j = 1 \leq i \leq n\}$ and

$E(L_n) = \{u_i v_j = 1 \leq i \leq n\}$

Define f : v(Ln) → {1, 2, …….. 5n-2} is defined as follows

$f(u_1) = 1,$

$f(v_i) = 5n\text{-}2\text{-}3h \quad 0 \leq h \leq n\text{-}1$

$f(u_2) = 2,$

$f(u_3) = 3,$

let f * be the induced edge labelsof f. Then

$f^*(u_i v_j) = 5n\text{-}2\text{-}1$

$f^*(u_i v_j) = f^*(f^*(u_{i-1} v_{j-1}) - (2+i) \qquad 2 \leq i \leq 3$

$f^*(u_1 u_2) = 5n\text{-}4$

$f^*(u_2 v_3) = 5n\text{-}10$

$f^*(u_1 v_2) = 5n\text{-}6$

$f^*(u_2 u_3) = 5n\text{-}9$

Then the induced edges are all. Distinct. Hence from the above labeling pattern, the graph $L_n$ admits super felicitous difference labeling graph.

**Example 3.5.1:** The SFD labeling graph of $\boldsymbol{l_3}$ is given in fig 4 respectively.

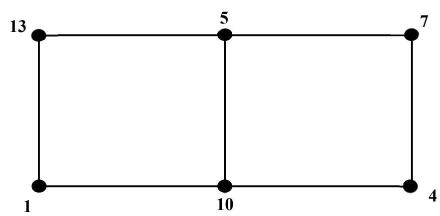



**Fig. 4**

**Case (ii):** Assume $n \leq 5$

Let $V(L_n) = \{u_i v_j / 1 \leq i \leq n, 1\}$ and

$E(L_n) = \{u_i v_j / 1 \leq i \leq n\}$

Define f: $v(L_n) \rightarrow \{1, 2, \ldots\ldots 5n - 2\}$ is defined as follows:

$f(u_i) = i \qquad 1 \leq i \leq 3$

$f(u_4) = 6$

$f(u_5) = 7$

$f(v_j) = 5n - 2$

$f(v_{2i}) = 5n - 5i \qquad 1 \leq i \leq 2$

$f(v_{2i+1}) = f(v_{(2i+1)-1} - 4) \quad 1 \leq i \leq 2$

let f* be the induced edge labels

$f^*(u_1 v_1) = 5n\text{-}2\text{-}i$

$f^*(u_{2i}, v_{2j}) = \{u_{2j+1}, v_{2j+1}) - 4, \qquad 0 \leq i \leq 1, 1 \leq i \leq 2$

$f^*(u_{2i+1}, v_{2j+1}) = \{u_{2i}, v_{2j}) - 5, \quad 0 \leq i \leq 1, 1 \leq i \leq 2$

$f^*(v_1, u_2) = 5n - 6$

$f^*(u_2, v_3) = f^*(v_1, u_2) - 2$

$f^*(v_3, u_4) = f^*(u_2, v_3) - 5$

f* ($u_4$, $v_5$) = f * ($u_3$, $v_4$) – 4
f* ($u_1$, $v_2$) = 5n – 4
f* ($v_2$, $u_3$) = f * ($u_1$, $v_2$) – 7
f* ($u_3$, $v_4$) = f * ($v_2$, $u_3$) – 4
f* ($v_4$, $u_5$) = f * ($u_3$, $v_4$) – 5

Then the induced edge labels are all distinct. Hence from the above labeling pattern, the graph $L_n$ admits SFDL graph.

**Example 3.5.2:** The SFD labeling graph of $\boldsymbol{l_5}$ is given in fig 5 respectively.

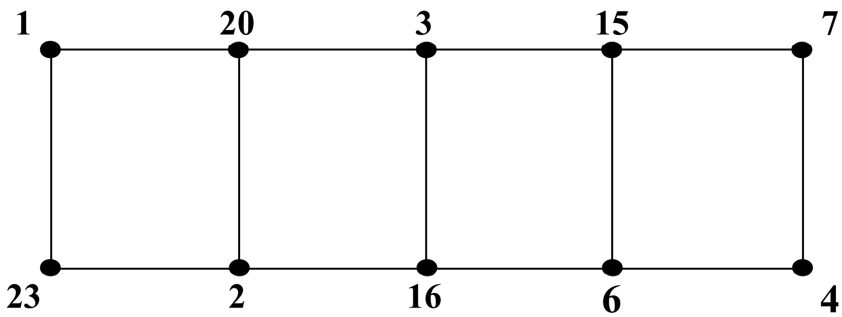



**Fig. 5**

**Theorem 3.6:** $H_{n,n}$is Super Felicitous Difference Labeling graph for all n.

**Proof:** Let V ($H_{n,n}$)= $\{u_i: 1 \leq i \leq n\} \cup \{v_j: 1 \leq j \leq n\}$and

E ($H_{n,n}$) =$\{\, u_i v_j: 1 \leq i \leq n \text{ } and \text{ } 1 \leq j \leq n\}$.

Define f: V ($H_{n,n}$) → $\left\{1,2, \ldots\ldots \frac{n^2+5n}{2}\right\}$ as follows

f($u_i$) = i $\quad 1 \leq i \leq n$

f($v_1$) $= \frac{n^2+5n}{2}$

f($v_{j+1}$) = $f\left(v_{j+1-1}\right) - (j+1), 1 \leq j \leq n$

Then the induced edge labels are all distinct. Hence from the above labeling pattern the graph admits $H_{n,n}$ SFDL graph.

**Example 3.6.1:** The SFD labeling graph of $H_{5,5}$ is given in fig 6 respectively.

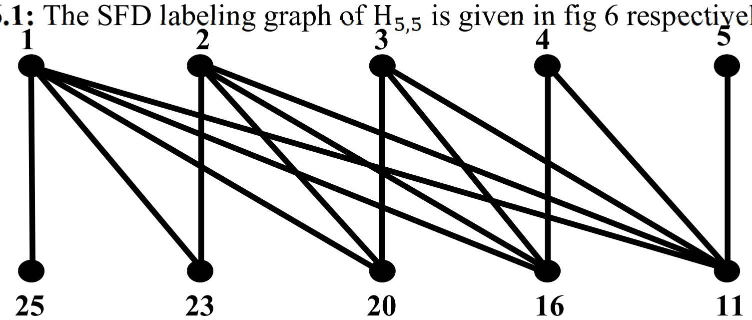



**Fig. 6**

**Theorem3.7:** The splitting graph of star Spl ($K_1$, n) is SFDL graph for all n.

**Proof: Let** V(G) = {u, v, $u_i$, $v_i$ / $1 \leq i \leq n$} and

E(G)= {$uu_i$, $uv_i$, $vv_i$ / $1 \le i \le n$}
Then │V (G)│=2n + 2, and │E (G)│ = 3n
Define f: V(G) → {1, 2, …….. 5n+2} as follows.
f (u) = 4n + 1
f ($u_i$) = n+i $1 \le i \le n$
f ($v_i$) = i $1 \le i \le n$
f (v) = 5n + 2
Letf $^*$ be the induced edge labeling of f. Then
$f^*$ ($uu_i$) = 3n – j $0 \le j \le n – 1$
$f^*$ ($vv_i$) = 5n + 1 - j $0 \le j \le n – 1$
$f^*$ ($uv_i$) = 4n - j $0 \le j \le n – 1$
Then the induced edge labels are all distinct. Hence from the above labeling pattern, the splittinggraphs admits SFDL graph.
**Example 3.7.1:** The SFD labeling graph of Spl($K_1$,4) is given in fig 7 respectively.

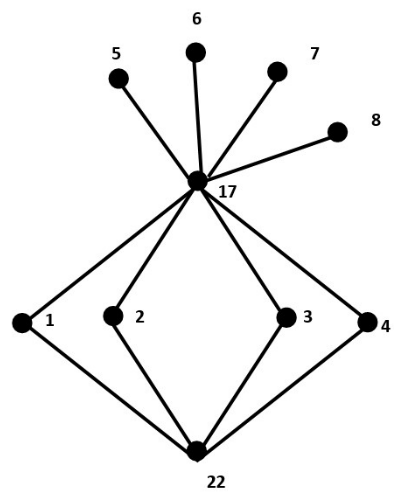



**Fig. 7**

**Theorem 3.8:** $P_2(+)N_{2n}$is a SFDL graph.
**Proof:** Let V ($P_2(+)N_{2n}$)= $\{u,v\} \cup \left\{{}^{u_i}/_{1 \le i \le n}\right\}$ and
E ($P_2(+)N_{2n}$) =$\{(u,v)\} \cup \{(u,u_i) \cup (v,u_i)/1 \le i \le n\}$.
Define f: V ($P_2(+)N_{2n}$) → $\{1,2, \ldots ..2n+2, 2n+3\}$ by
$f(u) = 3n + 3$ $f(v) = 2n + 2$
$f(u_i) = i$ $1 \le i \le n$
The induced edges are as follows
$f^*(uv) = n + 1$
$f^*(uu_i) = 3n + 3 - i,$ $1 \le i \le n$
$f^*(vu_i) = 2n + 2 - i,$ $1 \le i \le n$
Then the induced edge labels are all distinct. Hence from the above labeling pattern, the graph $P_2(+)N_{2n}$ admits SFDL graph.
**Example 3.8.1:** The SFD labeling graph of $P_2(+)N_8$are given in fig 8 respectively.

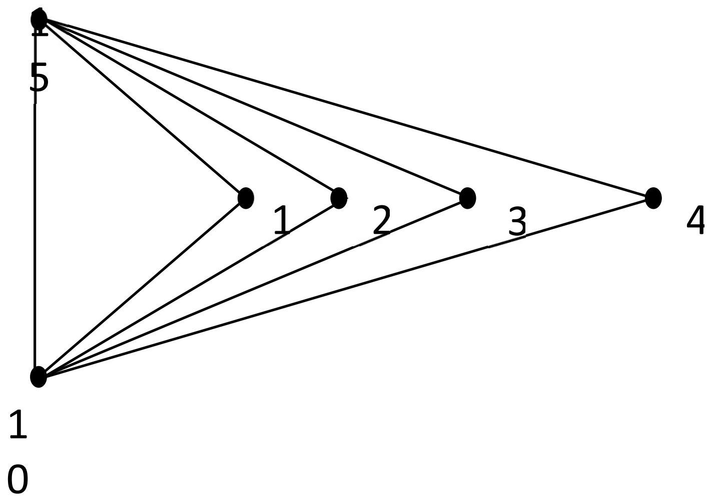



**Fig. 8**

**Theorem 3.9:**Jewel graph$J_n$is a SFDL graph.

**Proof:** Let V ($J_n$)= $\{u, v, x, y, w_i / 1 \leq i \leq n\}$ and

E ($J_n$) =$\{ux, vx, uy, vy, uw_i, vw_i / 1 \leq i \leq n\}$. Then │V (G)│=n + 4, and │E (G)│ = 2n+4.

Define f: V ($J_n$) → $\{1,2, \ldots .. 3n + 8\}$ by

$f(u) = 1$ $\quad$ $f(v) = 2$

$f(x) = 3n + 8$

$f(y) = 3n + 5$

$f(w_i) = f(y) - 3i \quad 1 \leq i \leq n$

Let $f^*$be the induced edge label of f. then

$f^*(ux) = 3n + 7$

$f^*(vx) = 3n + 6$

$f^*(vy) = 3n + 3$

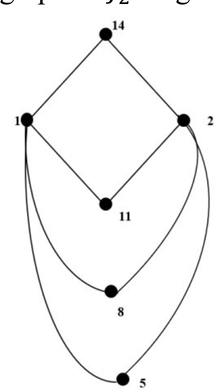
$f^*(uy) = 3n + 4$

$f^*(uw_i) = f^*(uy) - 3i, \quad 1 \leq i \leq n$

$f^*(vw_i) = f^*(vy) - 3i, \quad 1 \leq i \leq n$

Then the induced edge labels are all distinct. Hence from the above labeling pattern, the graph $J_n$ admits SFDL graph.

**Example 3.9.1:** The SFD labeling graph of $J_2$are given in fig 9 respectively.



## IV. Conclusion

In this paper, we investigated the Super Felicitous Difference Labeling of some special types of graphs. We have already investigated graphs which are SFDL graph only for certain cases[3]and have planned to investigate the SFD labeling of some special cases of cycle related graphs in our next paper.

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