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A NOVEL TECHNIQUE FOR IDENTIFYING DISTINCT INVERSIONS IN KINEMATIC CHAINS USING THE APPROACH OF LINK-JOINT-LOOP ADJACENCY

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Abstract
Mechanisms are always needed for any machine for imparting the motion and thus transmitting the power. Hence it is very essential to review the necessary characteristics of any mechanism in correspondence to synthesizing and analyzing the structural aspects of a kinematic chain. From the early 70's, many of the investigators had made their sincere efforts for the detection of kinematic chains with specified degrees of freedom that are distinct in nature, for verifying them with suitable isomorphism tests and also for identifying exact number of distinct mechanisms or inversions in a specified kinematic chain. Most of the techniques basically depends on studying and analyzing the two of the three basic parameters of a specified kinematic chain i.e joints and the links. But the existence of loops and its involvement in the motion capabilities of a mechanism is someway ignored. The widespread investigation and assessment of kinematic structure of any chain must be dependent on analyzing all three basic parameters i.e. joints, links and also loops. An attempt has been made in this paper to come up with a novel technique on the basis of composition of matrix for identifying the exact number of inversions that are distinct in nature for a specified kinematic chain. The suggested technique is proficient and trustworthy as it reflects the involvement of loops in a kinematic chain in addition to the joints and the links thereby using link-joint-loop adjacency. The technique had been applied to all the basic groups of kinematic chains and the results are verified with the results earlier reported.
Keywords: Distinct mechanism, Kinematic chain, Distinct Inversions, Adjacency matrix

1. Introduction

In designing of machine components and elements, analyzing the kinematic structure and synthesizing the kinematic chain is a major task and thus selecting a specific mechanism which can impart a predefined motion to the machine assembly plays a significant phase in the mechanism design. Various techniques and methodologies were suggested by past investigators in last few decades to structurally analyze the basic parameters of a kinematic chain. In this regards, identification of district inversions in a specified kinematic chain is a must and hence a vast analysis and research has been going on for developing techniques and methodologies which must be easy, proficient and trustworthy. During the start of 70's, the approach of graph theory was introduced for firstly identifying distinct inversions from the existing kinematic chains with configuration of nine links having two degree of freedom which resulted in the identification of 219 distinct inversions (Manolescu, 1973). Thereafter, a technique on the basis of using computerized technology was suggested which resulted in the identification of 71 discrete inversions for configuration of 8-links having 1-dof, 254 inversions for configuration of 9-links having 2-dof and also1834 inversions for configuration of 10-links having 1-dof kinematic chains (Mruthyunjaya, 1984). A novel technique was introduced later known to be MIN code and the approach was quiet significant in identifying the exact amount of inversions in the kinematic chain that are distinct (Ambekar et al., 1987). Detection of distinct mechanisms by sketching velocity diagrams and then by formulating a point for transfer of motion is also proposed later. (Patel et al., 1988). A chain of hamming string was formulated using the technique of hamming number which was used for reveling the accurate quantity of all the inversion that are possible from a given kinematic chain (Rao et al., 1991). For successfully detecting the mechanisms, the topological relationship between all the links of a kinematic chain were described by constructing a links adjacent chain table (Chu Jin Kui et al., 1994). Identification of distinct mechanisms is carried out by considering the degrees of the links as well as the variety of the joints along with the degree of freedom in a specific kinematic chain by proposing an innovative variant known as the sequential arrangement of all the links with regards to modified ranks of total distance (Yadav et al., 1996). Also identification of distinct mechanisms in a specified kinematic chain can be done directly by a scheme of pseudo probability while the technique was basically developed for discrete representation of kinematic chain (Sanyal et. al., 1997). Various other characteristics of a kinematic chain was detected which also involves identifying the distinct inversions on the basis of fuzzy logic, information theory and loop based techniques was suggested by Rao and coworkers (Rao et. al., 2000). Other kinematic characteristics such as inversions, freedom types, parallelism in chains as well as symmetry of chain along with discovering the isomorphism using the perception of correlation was presented (Srinath et. al., 2006). The enumeration of inversions of a kinematic chain on the basis of techniques of group theory was a novel approach and also a technique was introduced for specifying a kinematic chain with absence of isomorphism and chains that are degraded for various screw system (Simoni et al., 2009). Identifying discrete mechanisms in the existing planar kinematic chain can also be done by the development of two novel invariants for each of the links known as the first adjacency link value [FALV] and second adjacency link value [SALV] (Dargar et. al., 2009). Detection of isomorphism as well as the identification of inversions in kinematic chains by using the several joint connectivity's at diverse levels was suggested (Bedi et al., 2010). Later the concept of ability of motion transfer was introduced for detection of distinct mechanisms and identification of isomorphism and the type of the joint was also included which made the method more operative and also further improves the technique (Bedi et al., 2011). Madan proposed the invariant labeling systems for the links for detecting the inversions. Thereafter, the enumeration of all the probable distinct mechanisms from a specific kinematic chain, a distinctive algorithm was suggested by using the technique of adjacency matrices (Madan, 2014). The quantity of mechanisms for a specific kinematic chain with multiple or numerous joints can easily be determined by using the [JJ] matrices (Rizvi et al., 2016). Detection of both the inversions and the isomorphism can be done by the formation of strings of the chains as well as for the links along with the development of a least distance matrix (LDM) (Dewangan et al., 2019). Lastly, detection of discrete mechanisms is done by the development of an exclusive computational table known as the 'remote adjacency influence table' which controls the progression of the calculations of the adjacency and also the introduction of an algorithm was done known as the 'additive adjacency' which defines a novel framework (Kamesh et al., 2021). Most of the methodologies and approaches suggested by the previous investigators are based on indirectly identifying the distinct mechanisms or consumes an enormous amount of computational time and also their phases a way too complex. Moreover, all the mentioned techniques are solely dependent on any two of the three basic parameters of kinematic chain i.e. adjacencies of link vs joint, link vs loop or link vs link. Thus, for the identification of distinct inversions in a particular kinematic chain with single as well as multiple degrees of freedom, a quiet simple and highly trustworthy technique has been suggested which uses the involvement of all the three basic significant parameters of kinematic chain i.e loop, link and the joint. The technique is successfully applied to all the basic families of kinematic chain having single as well as multiple degrees of freedom taken from the past literatures and the outcomes are verified by the earlier reported outcomes.

2. Representation and Notation of a Planar Kinematic Chain

Three planar kinematic chain comprising of 8-links with 1-dof having combinations of binary, ternary as well as quaternary links having ten revolute joints is shown in Fig. 1.

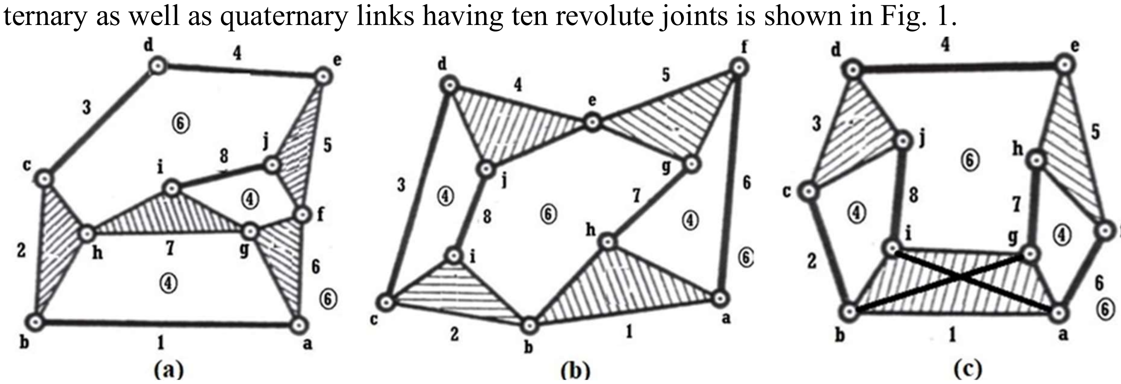

Figure 1: Three Kinematic Chains with Configuration of 8-Links 1-dof

The joints and the links are given notations usually by using the alphabets and numbers, respectively. The loops occurring in the chain are divided into three categories as independent loop (shortest loop), sub-loop (intermediate loop) and the outermost loop involving all the outer links (largest loop). The loop value is given to each of the loop according to the number of links occurring in the loop. Thus, it can be observed that the kinematic chain shown in Fig. 1(a) comprises of following loops:

Three independent loops: a-g-h-b-a (4), f-j-i-g-f (4) and c-d-e-j-i-h-c (6)
Two sub-loops: a-b-h-i-j-f-a (6) and c-d-e-f-g-h-c (6)
One outermost loop: a-b-c-d-e-f-a (6)

3. Link-Joint Values (LJV) and Link-Loop Values (LLV)

The link-joint value (LJV) creates a correlation between a specified link with all the joints occurring in the chain. Now, when a binary link joins with a ternary link, then it has three remaining joints which can be further connected to other links. Hence, the LJV for that joint will be computed as '3'. The link-1 (binary link) shown in Fig. 1(a) is connected to link-6 (ternary link) at joint 'a' and link-2 (ternary link) at joint 'b'. Hence, the LJV for '1-a' will be computed as '3' and also for '1-b' as '3'. All the other joints of the kinematic chain are at a particular distance from the specified link-1. Now, the LJV for these joints will be computed by considering the shortest distance of these joints from the joints of link-1 i.e 'a' and 'b'. It can be observed that join 'c' is at a distance of '2' units from 'b' and '3' units from 'a' and hence the shortest distance of 'c' with joints of link-1 is '2'. Hence, LJV for '1-c' is computed as '2'. Similarly, the LJVs for '1-d' is '3', for '1-e' is '3', for '1-f' is '2', for '1-g' is '2', for '1-h' is '2', for '1-i' is '3' and for '1-j' is '3'. Similar procedure is adopted for finding the correlation of all the remaining links with all the joints of the kinematic chain and the results in form of LJVs are tabulated in the matrix.

The link-loop value (LLV) creates a correlation between the link and the loop of a kinematic chain in accordance with the joint participating in that loop but is computed only for the joints of a specified link in consideration whereas for rest of the joints, LLV will be computed as zero. The LLV will be the summation of any of the two categories of loops in which the motion of the specified link is more effective. While observing a kinematic chain in consideration, the identification of both the categories of the loops which has to be taken for summation for computing LLV, following four cases has to be taken into account:

Case 1: When a kinematic chain is such that its structure comprises of only ternary and binary links, then LLV of an outer joint will be computed by the summation of participation of the joint in its independent loop as well as in the outermost loop. As observed in Fig. 1(a), the link-1 has an outer joint at 'a' which has its effective involvement in the independent loop a-g-h-b-a comprising of 4-links and also in the outermost loop a-b-c-d-e-f-a comprising of 6-links. Hence, LLV for '1-a' will be computed by their summation as '10'. Similarly, the LLV for '1-b' is also computed as '10'. The LLVs for all the other remaining joints in relation with the link-1 will be computed as zero.

Case 2: When a kinematic chain is such that its structure comprises of binary, ternary, quaternary or other higher order links, then LLV of an inner joint will be computed by the summation of participation of the joint in its two adjacent independent loops. As observed in Fig. 1(a), the link-8 has an inner joint at 'i' which has its effective involvement in the independent loop f-j-i-g-f comprising of 4-links and also in adjacent independent loop c-d-e-j-i-h-c comprising of 6-links. Hence, LLV for '8-i' will be computed by their summation as '10'. Similarly, the LLV for '8-j' is also computed as '10'. The LLVs for all the other remaining joints in relation with the link-8 will be computed as zero.

Case 3: When a kinematic chain is such that its structure comprises of only ternary and binary links and at any portion of the chain, two opposite binary links are in connection with two opposite ternary links, then they are said to be in 4-bar parallelism. For such a case, the LLV of an outer joint will be computed by the summation of participation of the joint in its independent loop as well as in the sub-loop. The LLV for inner joints will be computed referring to Case 2. As observed in Fig. 1(b), two opposite binary links i.e. link-3 and link-8 are in connection with two opposite ternary links i.e. link-2 and link-4 thereby creating a 4-bar parallelism. In this chain, link-1 has an outer joint at 'a' which has its effective involvement in the independent loop a-h-g-f-a comprising of 4-links and also in the sub-loop a-b-i-j-e-f-a comprising of 6-links. Hence, LLV for '1-a' will be computed by their summation as '10'. Similarly, the LLV for '1-b' is computed as '12'. The LLV for '1-h' is computed as '10' as per Case 2. The LLVs for all the other remaining joints in relation with the link-1 will be computed as zero.

Case 4: When a kinematic chain is such that its structure comprises of quaternary or quinary or other higher order links along with ternary and binary links, then the LLV of an outer joint will be computed by the summation of participation of the joint in its independent loop as well as in the sub-loop. The LLV for inner joints will be computed referring to Case 2. As observed in Fig. 1(c), link-1 has an outer joint at 'a' which has its effective involvement in the independent loop a-g-h-f-a comprising of 4-links and also in the sub-loop a-i-j-d-e-f-a comprising of 6-links. Hence, LLV for '1-a' will be computed by their summation as '10'. Similarly, the LLV for '1-b' is also computed as '10'. The LLV for '1-i' as well as '1-g' is computed as '10' as per Case 2. The LLVs for all the other remaining joints in relation with the link-1 will be computed as zero.

Similar procedure is adopted for finding the correlation of all the remaining links with all the loops of the kinematic chain and the results in form of LLVs are tabulated in the matrix.

4. Construction of Link-Joint-Loop Adjacency Matrix (LJLAM)

The LJVs and LLVs obtained from previous section are tabulated in a three dimensional matrix known as Link-Joint-Loop Adjacency Matrix (LJLAM) which develops a correlation between all the three significant parameters of a specified kinematic chain as shown in Table 1 constructed for the kinematic chain of Fig. 1(a).

Table 1: Link-Joint-Loop Adjacency Matrix for Figure 1(a)

		Joints										Link String Value
		a	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	
Links	**1**	3	3	2	3	3	2	2	2	3	3	**26**
		10	10	0	0	0	0	0	0	0	0	**20**
	2	2	3	3	2	3	3	2	4	2	3	**27**
		0	10	12	0	0	0	0	10	0	0	**32**
	3	3	2	3	2	2	3	3	2	3	3	**26**
		0	0	12	12	0	0	0	0	0	0	**24**
	4	3	3	2	2	3	2	3	3	3	2	**26**
		0	0	0	12	12	0	0	0	0	0	**24**
	5	2	3	3	2	3	4	2	3	2	3	**27**
		0	0	0	0	12	10	0	0	0	10	**32**
	6	3	2	3	3	2	4	4	2	2	2	**27**
		10	0	0	0	0	10	8	0	0	0	**28**
	7	2	2	2	3	3	2	4	4	3	2	**27**
		0	0	0	0	0	0	8	10	10	0	**28**
	8	3	3	3	3	2	2	2	2	3	3	**26**
		0	0	0	0	0	0	0	0	10	10	**20**

Now, the summation of all the respective LJVs of a link will result in the yielding of Link-Joint String Value (LJSV) for that particular link. Similarly, the summation of all the respective LLVs of a link will result in the yielding of Link-Loop String Value (LLSV) for that particular link. Hence, the Link String Value (LSV) of a particular link in relation with the Link-Joint Values (LJVs) and Link-Loop Values (LLVs) is calculated by using the relation shown below:

$$\text{Link String Value of Link} = \frac{\text{Link-Joint String Value of Link}}{\text{Link-Loop String Value of Link}} = \frac{\text{Summation of all Link-Joint Values of Link}}{\text{Summation of all Link-Loop values of Link}}$$

In terms of equation, we can write:

$$\mathbf{LSV(l)} = \frac{\mathbf{LJSV(l)}}{\mathbf{LLSV(l)}} = \frac{\sum_{\mathbf{i=a(l)}}^{\mathbf{n(l)}} \mathbf{LJV(i)}}{\sum_{\mathbf{i=a(l)}}^{\mathbf{n(l)}} \mathbf{LLV(i)}} \qquad \ldots(4.1)$$

where, l = Notation of Link ranging from 1 to m

m = Number of Links in a KC

and, n = Number of Joints in a KC

Now, the value of LSV for Link-1 in Table 1 is calculated by using Equa. 4.1 as,

For Link 1, $LSV_1 = \frac{LJSV_1}{LLSV_1} = \mathbf{\frac{26}{20}}$

Similarly, LSVs can be computed for all the other remaining links as shown in Table 1.

5. Identification of Distinct Inversions

The identification of distinct inversions depends on the identification of distinct links and thus the number of DIs in a specified kinematic chain equals the number of DLs present in the chain. In present technique, the link which is yielding a unique value of LSV will be considered as DL. Also, those links whose LSVs as well as values of LJVs and LLVs are similar will constitute one DL. It can be observed from Table 1 that link-1 and link-8 are yielding similar LSVs as:

$$LSV_1 = LSV_8 = \mathbf{\frac{26}{20}}$$

Similarly it can also be observed that $LSV_2 = LSV_5 = 27/32$, $LSV_3 = LSV_4 = 26/24$ and $LSV_6 = LSV_7 = 27/28$. Hence, the kinematic chain shown in Fig. 1(a) is having four distinct links and thus will constitute four distinct inversions.

Now, a suitable schematic presentation of DI has to be defined which will consist of following string:

1. Link-Joint String Value and Link-Loop String Value of respective DL confined in square brackets as [LJSV/LLSV]
2. Similar combined Link-Joint Value/Link-Loop Value occurring in the string of a specified DL, in descending order of Link-Joint Value, along with their respective frequencies, confined in curly brackets as {n(Max. LJV/LLV), n(Next Lower LJV/LLV), **……**, n(Min. LJV/LLV)}

Hence, the DI schematic for link-1 and link-8 can be written as,

DI Schematic$_{1,8}$ = [26/20] {2(3/10), 4(3/0), 4(2/0)}

The identified DIs along with their respective DI schematic presentation for Fig. 1(a) is shown in Table 2.

Table 2: Identification of DLs and DI Schematics of Fig. 1(a)

KC	DI	DL	DI Schematic
Fig. 1(a)	**4**	1,8	[26/20] {2(3/10), 4(3/0), 4(2/0)}
		2,5	[27/32] {1(4/10), 1(3/12), 1(3/10), 3(3/0), 4(2/0)}
		3,4	[26/24] {1(3/12), 1(2/12), 5(3/0), 3(2/0)}
		6,7	[27/28] {1(4/10), 1(4/8), 1(3/10), 2(3/0), 5(2/0)}

The suggested technique is successfully applied to all the basic family groups of kinematic chains as per literature sources and the quantity of inversions computed for each of the group matches with the earlier reported results as shown in Table 3.

Table 3: List of Distinct Inversions in Kinematic Chains

S. No.	Group of KCs	Distinct KCs	Distinct Inversions
1	6-bar with 1-dof	2	**5**
2	8-bar with 1-dof	16	**71**
3	9-bar with 2-dof	40	**254**
4	10-bar with 3-dof	98	**684**
5	10-bar with 1-dof	230	**1834**

The outcomes in the form of distinct inversions, distinct links and distinct inversion schematics for two groups of kinematic chains i.e. 6-bar with 1-dof and 8-bar with 1-dof is shown in Appendix I and II.

6. Conclusion

The identification of complete distinct inversions for a specified kinematic chain with given degree of freedom can be done by quantitative techniques that must be proficient, trustworthy and should consume lesser time period. With this in view, a novel technique is suggested which involves analyzing of all the three significant parameters of any kinematic chain i.e. loops, joints and the links, taking altogether, using the concept of link-joint-loop adjacency thereby developing a link-joint-loop adjacency matrix yielding link-string values for each link of a specified chain. Each distinct link identified in the matrix also identifies a distinct inversion in the chain. The technique is successfully incorporated to all the available groups of kinematic chain as per the literature sources and the outcomes are validated with the results reported earlier.

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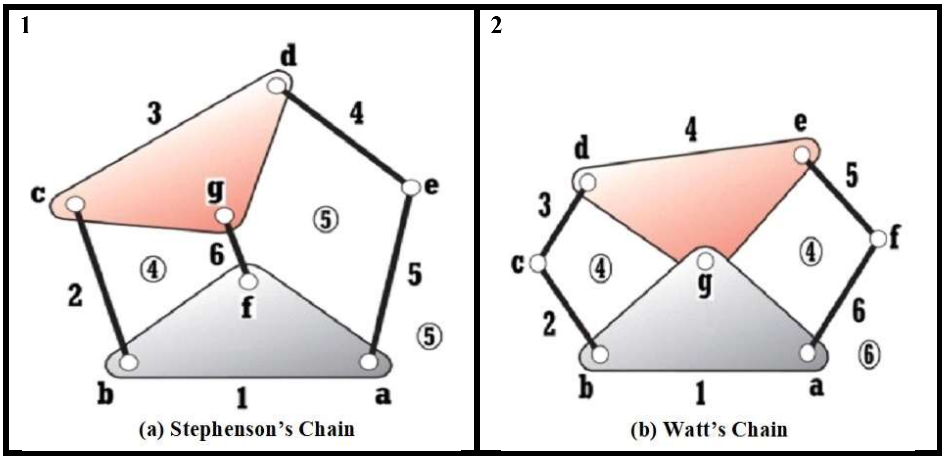
Appendix –I

Distinct Inversions in 6-links 1-dof KCs

KC	DI	DL	DI Schematic
Stephenson's Chain	**3**	1,3	[18/28] {1(3/10), 2(3/9), 1(3/0), 3(2/0)}
		2,6	[17/18] {2(3/9), 1(3/0), 4(2/0)}
		4,5	[17/20] {1(3/10), 1(2/10), 2(3,0), 3(2/0)}
Watt's Chain	**2**	1,4	[18/28] {1(4/8), 2(3/10), 4(2/0)}
		2,3,5,6	[17/20] {1(3/10), 1(2/10), 2(3/0), 3(2/0)}
Total Distinct Inversions = 5			



(a) Stephenson's Chain (b) Watt's Chain

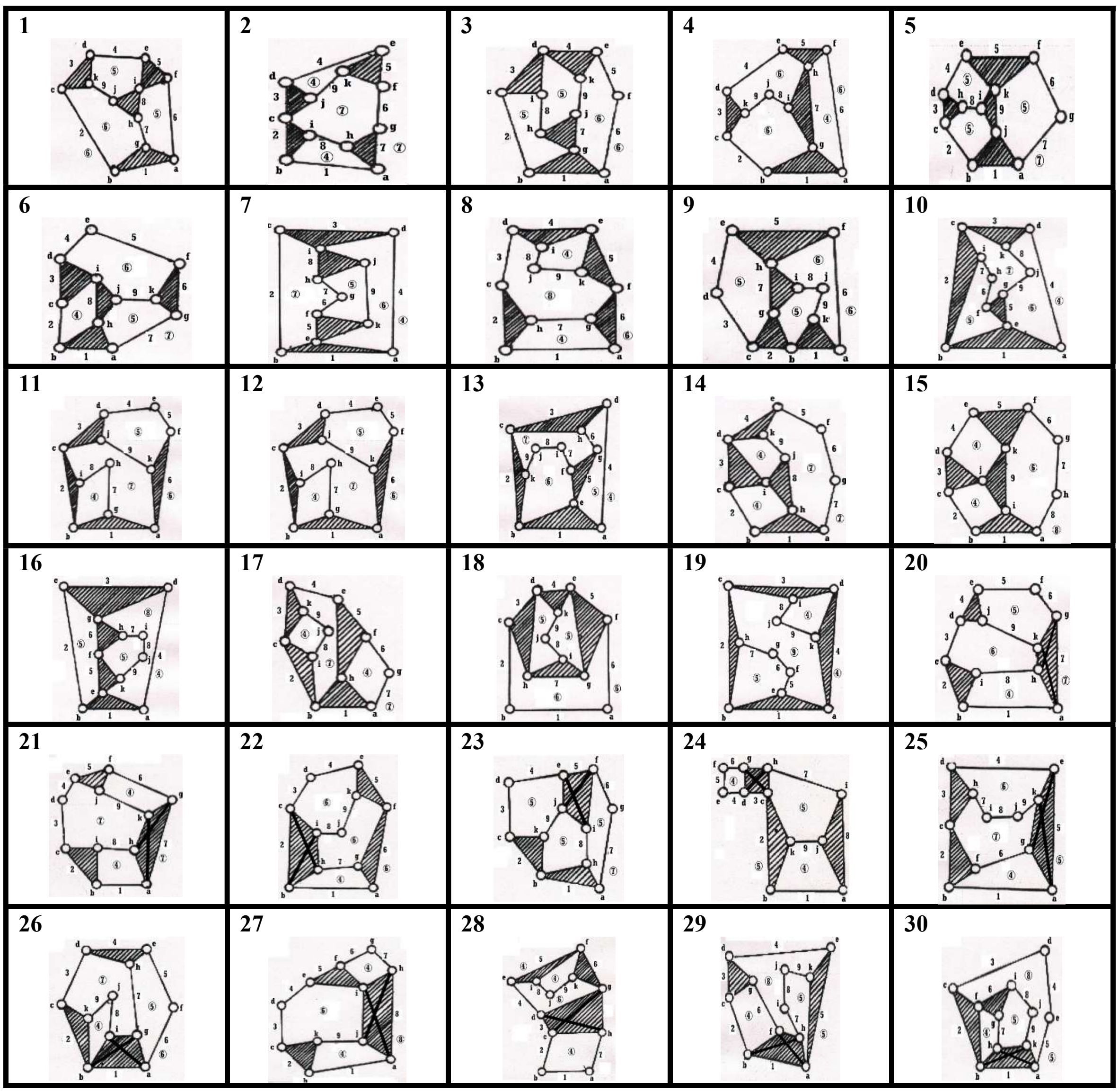
Appendix – II

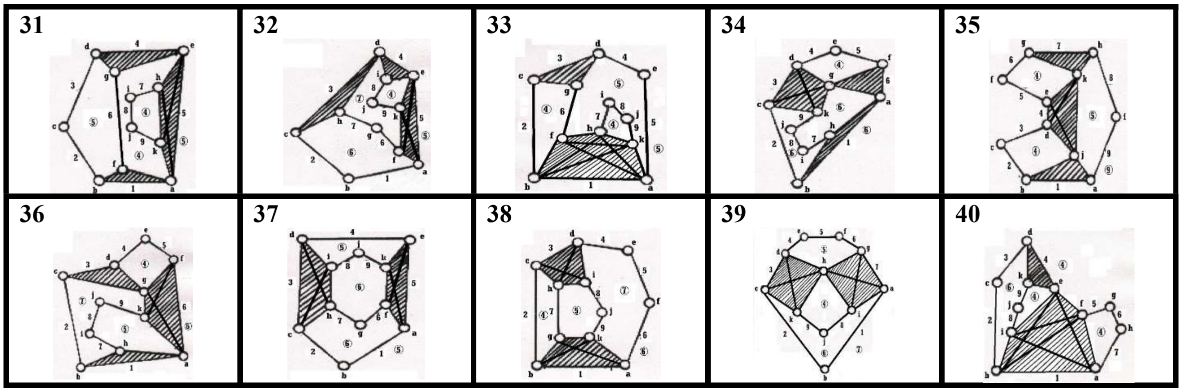
Distinct Inversions in 8-links 1-dof KCs

KC	DI	DL	DI Schematic
1	**2**	1,2,5,6	[27/32] {1(4/12), 1(4/8), 1(3/12), 2(3/0), 5(2/0)}
		3,4,7,8	[28/24] {1(3/12), 1(2/12), 2(4/0), 3(3/0), 3(2/0)}
2	**4**	1,8	[26/20] {2(3/10), 4(3/0), 4(2/0)}
		2,5	[27/32] {1(4/10), 1(3/12), 1(3/10), 3(3/0), 4(2/0)}
		3,4	[26/24] {1(3/12), 1(2/12), 5(3/0), 3(2/0)}
		6,7	[27/28] {1(4/10), 1(4/8), 1(3/10), 2(3/0), 5(2/0)}
3	**5**	1,3	[27/22] {2(3/11), 1(4/0), 3(3/0), 4(2/0)}
		2	[28/30] {1(4/8), 2(3/11), 1(4/0), 2(3/0), 4(2/0)}
		4,7	[27/32] {1(4/9), 1(3/12), 1(3/11), 3(3/0), 4(2/0)}
		5,6	[27/24] {1(3/12), 1(2/12), 1(4/0), 4(3/0), 3(2/0)}
		8	[27/26] {2(4/9), 1(4/8), 1(3/0), 6(2/0)}
4	**4**	1,8	[27/18] {1(4/0), 2(3/9), 3(3/0), 4(2/0)}
		2,7	[27/28] {1(4/10), 2(3/9), 3(3/0), 4(2/0)}
		3,6	[27/30] {1(4/10), 1(4/9), 1(3/11), 2(3/0), 5(2/0)}
		4,5	[28/22] {1(3/11), 1(2/11), 2(4/0), 3(3/0), 3(2/0)}
5	**8**	1	[27/28] {1(4/10), 1(4/9), 1(3/9), 2(3/0), 5(2/0) }
		2	[27/30] {1(4/10), 1(4/9), 1(3/11), 2(3/0), 5(2/0)}
		3	[26/22] {1(3/11), 1(2/11), 5(3/0), 3(2/0)}

KC	DI	DL	DI Schematic
		4	[27/22] {1(3/11), 1(2/11), 1(4/0), 4(3/0), 3(2/0)}
		5	[27/32] {1(4/10), 2(3/11), 3(3/0), 4(2/0)}
		6	[27/18] {2(3/9), 1(4/0), 3(3/0), 4(2/0)}
		7	[27/30] {1(4/10), 1(3/11), 1(3/9), 3(3/0), 4(2/0)}
		8	[26/22] {2(3/11), 4(3/0), 4(2/0)}
6	**3**	1,2,4,5	[26/20] {1(4/10), 1(2/10), 4(3/0), 4(2/0)}
		3,6	[30/38] {2(4/10), 2(4/9), 2(3/0), 4(2/0)}
		7,8	[26/18] {2(4/9), 2(3/0), 6(2/0)}
7	**2**	1,2,5,8	[27/30] {2(4/10), 1(3/10), 2(3/0), 5(2/0)}
		3,4,6,7	[27/20] {1(3/10), 1(2/10), 1(4/0), 4(3/0), 3(2/0)}
8	**5**	1	[30/36] {2(5/8), 2(4/10), 6(2/0)}
		2,8	[26/20] {1(4/10), 4(3/0), 1(2/10), 4(2/0)}
		3,7	[28/20] {1(3/10), 1(2/10), 2(4/0), 3(3/0), 3(2/0)}
		4,6	[27/28] {1(5/8), 2(3/10), 2(3/0), 5(2/0)}
		5	[26/24] {2(3/12), 4(3/0), 4(2/0)}
9	**8**	1	[30/37] {1(5/8), 2(4/10), 1(4/9), 1(3/0), 5(2/0)}
		2	[27/20] {1(4/10), 1(2/10), 1(4/0), 3(3/0), 4(2/0)}
		3	[27/20] {1(3/10), 1(2/10), 1(4/0), 4(3/0), 3(2/0)}
		4	[27/27] {1(5/8), 1(4/9), 1(3/10), 1(3/0), 6(2/0)}
		5	[27/28] {1(4/9), 1(3/10), 1(3/9), 3(3/0), 4(2/0)}
		6	[26/20] {1(3/10), 1(2/10), 1(4/0), 3(3/0), 4(2/0)}
		7	[26/20] {1(4/10), 1(2/10), 4(3/0), 4(2/0)}
		8	[26/18] {1(4/9), 1(3/9), 3(3/0), 5(2/0)}
10	**7**	1	[30/38] {1(5/9), 1(4/11), 2(4/9), 1(3/0), 5(2/0)}
		2,8	[26/18] {1(4/9), 1(3/9), 3(3/0), 5(2/0)}
		3	[28/30] {1(3/12), 2(3/9), 1(4/0), 3(3/0), 3(2/0)}
		4	[26/22] {1(3/12), 1(3/10), 4(3/0), 4(2/0)}
		5	[27/31] {1(5/9), 1(3/12), 1(3/10), 2(3/0), 5(2/0)}
		6	[28/22] {1(3/11), 1(2/11), 2(4/0), 3(3/0), 3(2/0)}
		7	[27/22] {1(4/11), 1(2/11), 1(4/0), 3(3/0), 4(2/0)}
11	**7**	1	[30/39] {1(4/11), 1(4/10), 2(4/9), 2(3/0), 4(2/0)}
		2	[26/22] {1(4/11), 1(2/11), 4(3/0), 4(2/0)}
		3	[27/22] {1(3/11), 1(2/11), 1(4/0), 4(3/0), 3(2/0)}
		4	[27/32] {1(4/11), 1(3/11), 1(3/10), 3(3/0), 4(2/0)}
		5	[27/29] {1(4/11), 2(3/9), 3(3/0), 4(2/0)}
		6,7	[26/18] {1(4/9), 1(3/9), 3(3/0), 5(2/0)}
		8	[26/20] {1(4/10), 1(3/10), 3(3/0), 5(2/0)}
12	**2**	1,2,4,5,7,8	[26/20] {1(4/10), 1(2/10), 4(3/0), 4(2/0)}
		3,6	[30/38] {1(6/8), 3(4/10), 6(2/0)}
13	**4**	1	[30/40] {4(4/10), 2(3/0), 4(2/0)}
		2,6,7,8	[26/20] {1(4/10), 1(3/10), 3(3/0), 5(2/0)}
		3,5	[27/32] {1(3/12), 2(3/10), 4(3/0), 3(2/0)}
		4	[26/24] {2(3/12), 4(3/0), 4(2/0)}
14	**2**	1,2,4,5	[27/32] {1(4/12), 2(3/10), 3(3/0), 4(2/0)}

KC	DI	DL	DI Schematic
		3,6,7,8	[26/20] {2(3/10), 4(3/0), 4(2/0)}
15	6	1	[27/20] {2(3/10), 1(4/0), 3(3/0), 4(2/0)}
		2,6	[27/30] {1(4/9), 1(3/11), 1(3/10), 3(3/0), 4(2/0)}
		3,5	[26/22] {2(3/11), 4(3/0), 4(2/0)}
		4	[27/32] {2(3/11), 1(3/10), 4(3/0), 3(2/0)}
		7	[27/28] {2(4/9), 1(3/10), 2(3/0), 5(2/0)}
		8	[26/20] {2(3/10), 4(3/0), 4(2/0)}
16	2	1,3,4,7	[27/30] {1(4/10), 2(3/10), 3(3/0), 4(2/0)}
		2,5,6,8	[26/20] {2(3/10), 4(3/0), 4(2/0)}
Total Distinct Inversions = 71			

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