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A FRACTIONAL-ORDER APPROACH FOR MODELLING ACTIVE LOW-FREQUENCY FILTER

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Abstract:

In this work, a low-frequency filter is taken into consideration, where the integer order capacitors and inductor are replaced with fractional-capacitor and general impedance converter (GIC) based fractional-inductor. The performance analysis of the fractional-order low-frequency filter (FOLFF) has been carried both in frequency and time domain. sensitivity analysis of the proposed filter has also been studied to explore the dependence of the filter parameter on exponent variations. It can be observed that, the incorporation of fractional-order elements increases the degree of design freedom of the FOLFF. On the other hand, the comparative analysis of fractional-capacitances with the different peak frequency, cut-off frequency and oscillatory frequency at various orders are obtained in simulation level.

Keywords: Fractional-order low-frequency filter (FOLFF), frequency-domain analysis, step-response, sensitivity analysis, fractional-capacitors, and general impedance converter (GIC).

1. Introduction

The quest for human power over the universe has grown in tandem with the advancement of knowledge and technology. This prompted researchers to look at the natural world around him in order to improve his console. The production of a vast range of electronic gadgets with a limitless number of applications has resulted in advancements in innovation. Among all of these devices, the filter is important in a variety of engineering fields, including control systems [1], biomedical engineering [2-3], and signal processing [4-6]. and image processing, among other things [7]. Operational amplifiers [8], current conveyors [9], current feedback amplifiers [10], operational trans-conductance amplifiers [11, 12].

Bioelectronics is currently attracting a lot of attention from researchers [13]. Bioelectronic signals are naturally low frequency signals, ranging from a few hertz to a few kilohertz, necessitating low cut-off frequency filters. Several approaches for implementing very low frequency filters have been proposed [14-17]. Low frequency filter design is important in a variety of applications, including medical, supply harmonics, control systems, and audio amplifiers. It's not easy to design a low pass filter with a cut off frequency of 100 Hz. Inductor works well in the design of an active filter at high frequencies, on the other hand practical inductors cannot be produced in low frequency applications within the region of (0-100 kHz) because [10]. Inductors are also challenging to produce in monolithic integrated circuits due to their huge size and weight. Biomedical signals are frequently in the 10mHz to 100Hz frequency range, necessitating the use of sub hertz frequency filters before processing. At high frequencies, the performance of passive filters begins to deteriorate. Several design ideas for achieving varied low cut-off frequency filters have already been reviewed:

In [19], a low frequency analog high pass filter with a cut off frequency of 0.1 Hz has been demonstrated utilizing tunable pseudo resistors. For medical and other low frequency applications, an active low frequency low pass filter with a cut off frequency of 100Hz is developed using a frequency dependent negative resistance (FDNR) as an alternative to inductors. The intrinsic property of fractional-order filters is that the pole frequency can be scaled down to sub-hertz by applying the order if the filter's normal pole frequency is slightly less than the one specified in. With cut-off frequencies up to 2 mHz and a power consumption of 5 nW, offers a first order low pass filter for signal conditioning applications, specifically for use with very low frequency physiological signals in low power portable medical equipment. Low-frequency sounds produced by interference from infrared gravity waves in wideband seismic signals are filtered out in the 1 Hz frequency range. This describes how to build fractional order active low frequency filters with operational amplifiers that can operate in the low frequency range in details.

The work done in this paper can be summarized as follows:

(a) The classical components of the low frequency filter are replaced by fractional-capacitor and GIC based fractional-inductor. The mathematical modelling of FOLFF is carried out and the transfer function and stability equations are determined numerically.

(b) The simulation has been performed to obtain the frequency-response, stability analysis as well as comparative analysis.

(c) The sensitivity analysis of the proposed FOLFF is carried out and the gain sensitivity, transfer function sensitivity, Pole Quality factor (Q) sensitivity and Pole frequency (ω_0) sensitivity.

2. Materials and methods

For achieving low cut-off frequency, the proposed filter consists of two Fractional order elements (FOE) as shown in figure 1. The FOE1 is a single component fractional order element as discussed in [9, 10] with order α which is less than one i.e., $0 \leq \alpha \leq 1$. However, FOE 2 uses a generalized impedance converter (GIC) circuits in which a single component fractional order element is used to get order β which is greater than 1 i.e., $1 \leq \beta \leq 2$. Single component FOE with $\beta \geq 2$ still could not be realized.

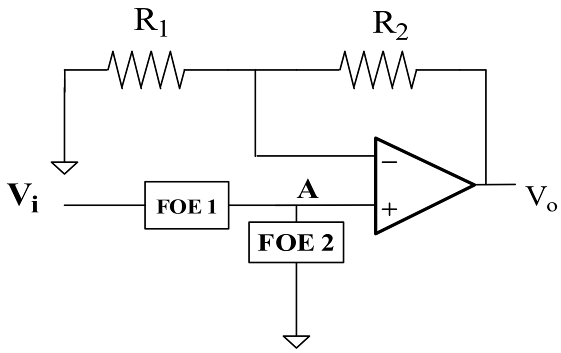


Figure 1 Circuit diagram of low frequency fractional order filters.

The order of FOE2 has been selected above one to get higher roll off factor. The GIC circuits used as FOE2 are shown in figure 2. Here Z_2, Z_5 are two resistors of value R, and Z_1, Z_3 are impedances of two capacitance i.e., $Z_1 = Z_3 = \frac{1}{C_2 s^{\beta}}$. Similarly, Z_4 is the impedance of a two terminal fractional order element i.e., $Z_4 = \frac{1}{C_2 s^{\gamma}}$.

(1)

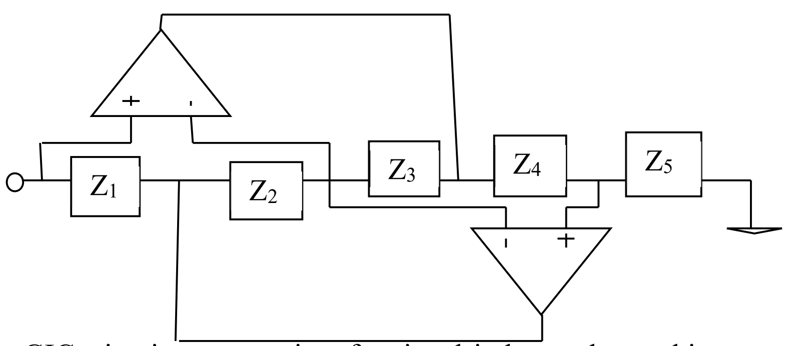


Figure 2. GIC circuit representing fractional-inductor by making use of fractional capacitor

As in [12], the impedance of GIC circuit is expressed as

$$Z = Z_{FOE2} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} = \frac{1}{C_2 s^{2-\gamma}} = \frac{1}{C_2 s^{\beta}} \quad (2)$$

This implies that the impedance Z of GIC is the impedance of a FOE [13, 14] whose order is β i.e. , $\beta = 2 - \gamma$. Similarly, since FOE1 is a two terminal fractional order element, whose impedance can be expressed as

$$Z_{FOE1} = \frac{1}{C_2 s^{\alpha}}$$

(3)

Now, the expression for output,

$$V_o(s) = \frac{K\eta}{s^{\beta-\alpha}+\eta} V_i(s), (\omega \neq 0) \tag{4}$$

where, $\eta = \frac{C_1}{C_2}$ and gain of filter, $K = 1 + \frac{R_2}{R_1}$

It may be observed from equation (3) that the filter parameter (peak frequency, gain cross-over frequency, cut-off frequency) depends on the ratio of two fractional capacitances, not on the value of capacitance and hence a lower order filter with low cut-off frequency/bandwidth can be realized. However, at DC (ω=0) the steady state output, $V_o = 0$.

Transfer function: From equation (2) and (3), transfer function of the proposed filter can be found and given as

$$T(s) = \frac{K\eta}{s^{\beta-\alpha}+\eta} \tag{5}$$

From transfer function, it is clear that the figure (1), act as low pass low frequency fractional order filter.

Stability: The stability of the filter can be analyzed in frequency domain by observing pole-zero location in S-plane and found that, system will stable if and only if η> 0 and (β - α) < 2, while it will oscillate if and only if η> 0 and (β - α) = 2, otherwise it is unstable [15]. Since the ratio of two fractional capacitance (η) is always positive and (β - α) is always less than two and +ve, the filter is always stable.

The location of pole is important to determine the filter center frequency (ω_o) and its quality factor Q. From equation (5), for 1 < (β - α) < 2, there are only two poles located at $s_{1,2} = \eta^{\frac{1}{\beta-\alpha}} e^{\pm j\frac{\pi}{\beta-\alpha}}$. Comparing with a classical second-order system whose poles is located $s_{1,2} = -\frac{\omega_o}{2Q} \pm j\omega_o\sqrt{1-(\frac{1}{4Q^2})} = \omega_o e^{\pm j\delta}$, Where, $\delta = cos^{-1}(-\frac{1}{2Q})$It can be seen that,

$$\omega_o = \eta^{\frac{1}{\beta-\alpha}}, Q = -\frac{1}{2\,co\,\frac{\pi}{(\beta-\alpha)}} \tag{6}$$

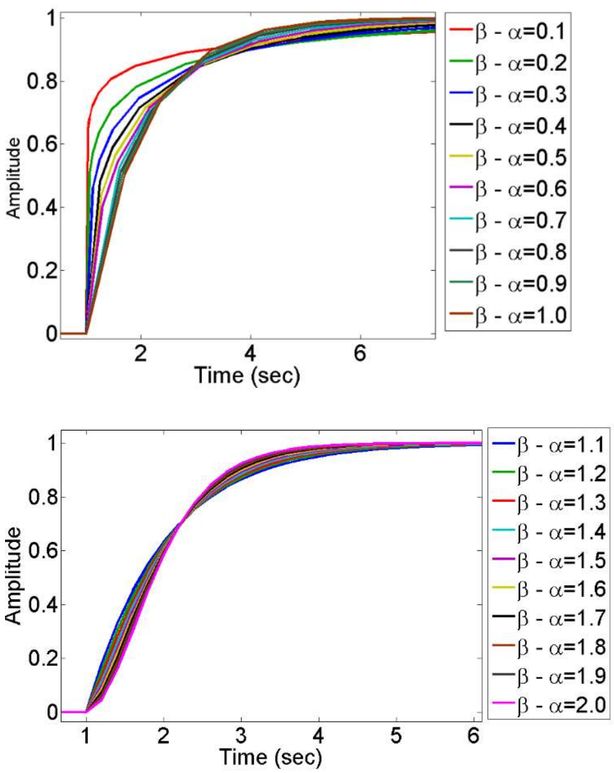
3. Simulations and Results of the proposed filter

3.1. Stability analysis

Considering the transfer function of the proposed fractional order filter, the stability analysis is being determined, Where, the fractional order transfer function is given by,

$$G(s) = \frac{1+\frac{R_2}{R_1}}{1+\frac{FC_2}{FC_1}s^{\beta-\alpha}} \tag{7}$$

Here the fractional orders α and β are dependable operators. As the fractional power of the Laplace operator is termed as β-α and α varies from 0.1 to 1.0 and β varies from 0.1 to 2.0, there are two outcomes, β-α>1 and β-α<1.



(a) (b)

Figure 3 Step response of proposed filter with orders (a) β-α≤1, and (b) β-α≤1.

Here, in the fig. 3 the step analysis at different fractional order combination β-α is shown. Where, β-α≤1. It is resulted that there is a marginal decrease in rise time and a distinguish increase in settling time on decreasing the order from 1.0 to 0.1. Again, on realizing the order $1 \leq \beta - \alpha \leq 1.9$ it is observed that, there is a very little change in the step response when the fractional orders are altered from 1.0 to 2.0. But it can be observed that at order $\beta - \alpha = 1.9$, a very low settling time is being observed, which justifies it to be a more stabilized system.

Table. 1 Step response parameters of proposed filter with orders β-α≤1.

β-α	Rise-Time(t_r) (sec)	Settling-Time(t_s) (sec)
0.1	1.03	24.78
0.2	1.22	16.83

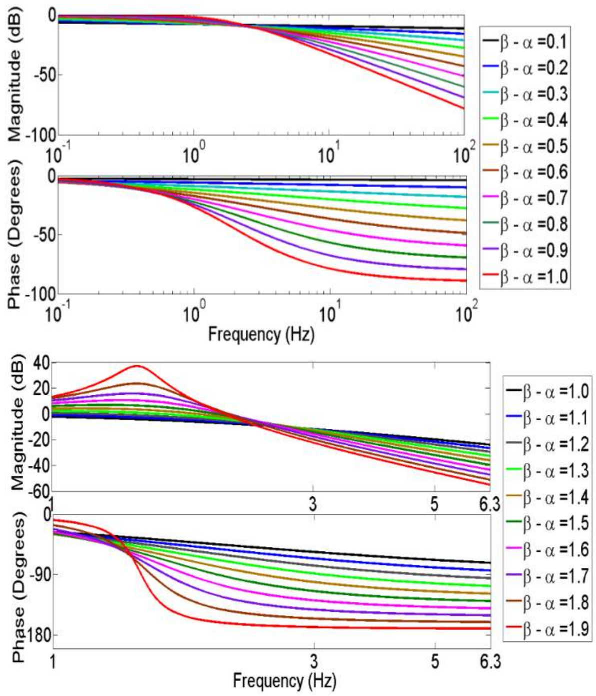
0.3	1.47	16.92
0.4	1.48	16.94
0.5	1.63	17.08
0.6	1.68	15.15
0.7	1.71	15.08
0.8	1.73	13.13
0.9	1.78	12.18
1.0	1.81	11.26

Table. 2 Step response parameters of proposed filter with orders β-α≥1.

β-α	Rise-time(t_r) (sec)	Settling-time(t_s)(sec)
1.1	1.03	24.78
1.2	1.22	16.83
1.3	1.47	16.92
1.4	1.48	16.94
1.5	1.63	17.08
1.6	1.68	15.15
1.7	1.71	15.08
1.8	1.73	13.13
1.9	1.78	12.18

3.2 Frequency Domain analysis

Now, the proposed fractional-order filter is analysed in frequency domain. Here, two cases are studies where the dependencies of fractional-orders α and β are seen, i.e., β-α≤1, and β-α≥1. The transfer function of the fractional-order low frequency filter is designed in MATLAB coding the frequency response were generated as shown in Fig.4 (a) and (b).



(a) (b)

Figure 4 Frequency-response of proposed filter with orders (a) β-α≤1, and (b) β-α≥1.

Here, in Fig.6 it is observed that, with change in fractional-orders the roll-off-rate varies significantly. It can be noted that for β-α≤1 the magnitude of roll of rate decreases and for β-α≥1 the magnitude of roll-off-rate increases. This can be mathematically, satisfied as,

$$Roll-off-rate = -20\alpha \; dB/decade \qquad (8)$$

Where, α is the fractional-order. So, with increase in the fractional-order value the magnitude of roll-off-rate increases. The design parameters data corresponding to Fig. 4 is well illustrated in Table. 3 and Table. 4.

Table. 3 Frequency-domain parameters of proposed filter with orders β-α≥1.

β-α	Centre-frequency(Hz)	Cut-offfrequency(Hz)	Roll-off-rat(dB/decade)
0.1	1.03	24.78	2
0.2	1.22	16.83	4
0.3	1.47	16.92	6
0.4	1.48	16.94	8
0.5	1.63	17.08	10
0.6	1.68	15.15	12
0.7	1.71	15.08	14
0.8	1.73	13.13	16
0.9	1.78	12.18	18
1.0	0.6	0.9	20

Table. 4 Frequency-domain parameters of proposed filter with orders β-α≥1.

β-α	Centre-frequency(Hz)	Cut-offfrequency(Hz)	Roll-off-rat(dB/decade)
1.1	1.03	24.78	22
1.2	1.22	16.83	24
1.3	1.47	16.92	26
1.4	1.48	16.94	28
1.5	1.63	17.08	30
1.6	1.68	15.15	32
1.7	1.71	15.08	34
1.8	1.73	13.13	36
1.9	1.78	12.18	38

3.3. Comparative analysis

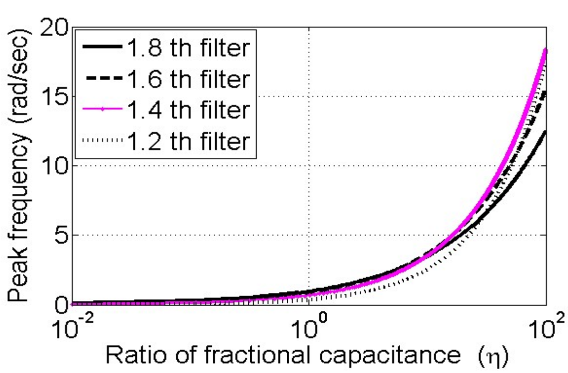
The filter parameters (peak frequency, gain cross-over frequency, cut-off frequency, and band width) of the proposed filter can be obtained by taking the magnitude of the transfer function. Peak frequency, $\boldsymbol{\omega_m}$(the frequency at which the magnitude response has a maximum or a minimum) can obtained by solving the equation $(d/d\omega)|T\ (j\omega)|\omega = \omega m\ =\ 0$ and found to be

$$\omega_m = \eta^{\frac{1}{\beta-\alpha}}(-\cos\frac{\pi(\beta-\alpha)}{2})^{\frac{1}{\beta-\alpha}} \quad (9)$$

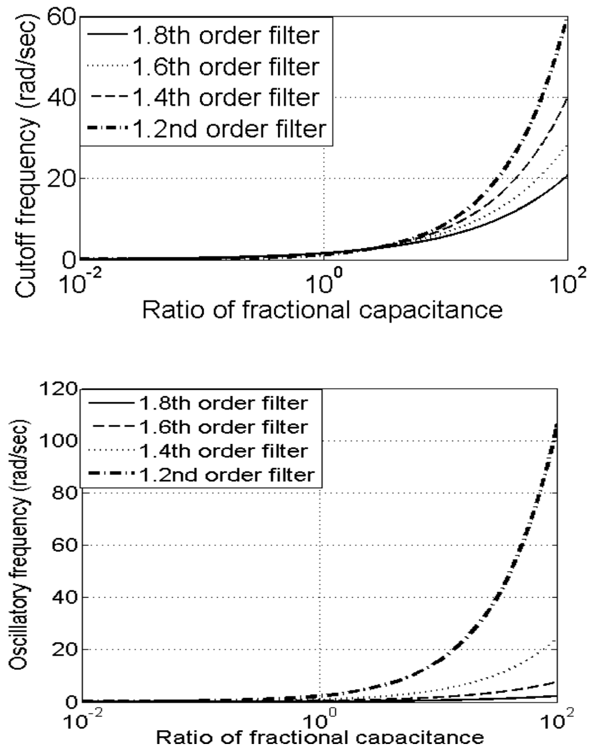
Similarly, the 3-dB frequency, ω_c (the frequency at which the magnitude response drops to 0.707 of pass band response) can be obtained by solving the equation $|T\ (j\omega h) = (1/\sqrt{2})|T\ (passband)|$ it is found to be,

$$\omega_c = \eta^{\frac{1}{\beta-\alpha}}[\sqrt{(1+\cos^2\frac{\pi(\beta-\alpha)}{2})} - \cos\frac{\pi(\beta-\alpha)}{2}]^{\frac{1}{\beta-\alpha}} \quad (10)$$

From fig. 5 (a), it is observed that for lower fractional capacitance ratio, the peak frequency increases and when order of filter increases peak frequency decreases when order increases. This is due to nonlinear relation between peak frequency and fractional capacitance ratio.



(a)



(b) (c)

Figure 5 Simulation of (a) peak frequency versus fractional capacitance, (b) cut-off frequency versus fractional capacitance ratio, and (c) oscillatory frequency versus fractional capacitance ratio.

From Fig. 5(b), it is observed that cut off frequency increase when ratio of fractional capacitance increases. Now the frequency at which the filter transfer function become purely imaginary or the filter will be oscillatory condition, can be obtained by equating phase of the transfer function equal to π/2 and found as seen in (9).

$$\omega_{rp} = \frac{\eta^{\frac{1}{\beta-\alpha}}}{(-\cos\frac{\pi(\beta-\alpha)}{2})^{\frac{1}{\beta-\alpha}}} \quad (11)$$

From Fig. 5(c), it is concluded that oscillatory frequency highly increases when ratio of fractional capacitance increases.

4. Sensitivity analysis of the proposed low frequency filter

As per the previous section, the sensitivity analysis of the low frequency fractional order filter can be done to investigate 'how much the filter's behavior changes as a component value changes. Now, pole frequency sensitivity, quality factor sensitivity and transfer function sensitivity can be obtained from equation (4), (5).

4.1 Gain sensitivity

Gain sensitivity can be calculated from equation (3a) using fundamental principle.

$$S_{C_1}^{K} = S_{C_2}^{K} = 0 \quad (12)$$

However, when $R_2 = R_3$, then

$$S_{R_2}^{K} = -S_{R_1}^{K} = \frac{R_2}{R+R_2} \quad (13)$$

However, when $R_2 = R_3$, then

$$S_{R_2}^{K} = -S_{R_1}^{K} = \frac{1}{2} \quad (14)$$

Hence, gain sensitivity depends on the variation of components R_1 and R_2. However, it is independent of other component variations.

4.2 Pole frequency (ω_o) sensitivity

Applying fundamental principle to the expression of pole frequency (ω_o) given in equation (6), we get

$$S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = \frac{1}{\beta-\alpha} \quad (15)$$

$$S_{\alpha}^{\omega_0} = -S_{\beta}^{\omega_0} = \frac{\beta}{(\beta-\alpha)^2} ln\,\eta \quad (16)$$

$$S_{R_2}^{\omega_0} = S_{R_1}^{\omega_0} = 0 \quad (17)$$

It is observed that pole frequency sensitivities towards C_1 and C_2 are inversely proportional to the fractional order of filter and hence, a higher order filter can reduce pole frequency variation. However, ω_o sensitivity towards R_2, and R_1 is zero which shows that these components variation does not have any impact on pole frequency. Similarly pole frequency sensitivities will reduce when FOE with lower exponent factor is used for realizing low frequency filter.

4.3 Pole Quality factor (Q) sensitivity

Applying fundamental principle to the expression of quality factor (Q) given in equation (6), we get,

$$s_{c_1}^{Q} = -s_{\alpha}^{Q} = \frac{\pi}{\beta-\alpha} \tan\left(\frac{\pi}{\beta-\alpha}\right) \quad (18)$$

$$s_{c_1}^{Q} = -s_{c_2}^{Q} = 0 \quad (19)$$

This shows that quality factor (Q) sensitivity does not depend on the value of the components, however depends on the order of filter.

4.4 Transfer function sensitivity

Transfer function sensitivities (i.e., changes in an entire transfer function relative to a component variation) are more complex and thus difficult to put to use. Basically, they are the functions of frequency and component values. Hence even if the component variation is minimized, the transfer function sensitivity cannot be zero when frequency is changing. Applying fundamental principle to the expression of transfer function given in equation (5), we can derive the sensitivity of transfer function towards various components, i.e., sensitivity towards α, β, C_1 and C_2 are given as

$$S_{\beta}^{T_{LP}(s)} = \frac{\beta\, l\ \ (s) s^{\beta-\alpha}}{s^{\beta-\alpha}+\eta} \quad (20)$$

$$S_{\alpha}^{T_{LP}(s)} = -\frac{\alpha\, l\ \ (s) s^{\beta-\alpha}}{s^{\beta-\alpha}+\eta} \quad (21)$$

$$S_{C_1}^{T_{LP}(s)} = S_{C_2}^{T_{LP}(s)} = \frac{s^{\beta-\alpha}}{s^{\beta-\alpha}+\eta} \quad (22)$$

5. Conclusions

In this paper, a new family of fractional order low-pass filters has been proposed. A very low frequency, going down to sub-Hertz cut-offs, low pass filter topology has been presented showing excellent characteristics. It allows to tuning of the filter at low cut off frequency and that of the filter attenuation between 0 and -6 dB per octave, separately and independently from one another. Also, the frequency domain analysis and step analysis has been carried out of the proposed fractional-order low frequency filter. These determine various design parameters like speed of response, maximum peak, cut-off frequencies and roll-of-rate. In fractional-domain a greater degree of freedom of these design parameters are experienced in simulation level. In practice, a simulation has been obtained using poles of filters, the poles of which exclusively depend on the desired cut off frequency, and using a linear combination of their outputs, the coefficients of which exclusively depend on the desired filter attenuation. This structure guarantees the stability even for time-varying parameters.

Conflict of interest

There is no conflict of interest among the authors.

References

1. D. J. Comer, D. T. Comer, B. K. casper and J. R. Gonzale, "An integrable single-pole low pass filter for low-frequency operation", International Journal of Electronics, Vol. 83:1, 49-54, DOI: 10.1080/002072197135634.

2. U. Yodprasit; J. Ngarmnil, "Micropower transconductor for very-low frequency filters", IEEE APCCAS 1998 Conference, 1998, **ISBN:**0-7803-5146-0,**DOI:** 10.1109/APCCAS.1998.743644.

3. J. Lee, M. D. Johnson and D. R. Kipke, "A tunablebiquad switched capacitor amplifier-filter for neural recording", IEEE transactions on biomedical circuits and systems, vol. 4, no. 5, October 2010.

4. M. Shayegannia, H. Al-Nashash,"Low Frequency Filter design using Gyrator for Biomedical applications", Department of Electrical Engineering – American University of Sharjah.

5. Y. A. H. Mahnashi, D. H. Abdullah Alzaher, Patent No.: US 8,866,542, Date of Patent: Oct. 21, 2014 "Low Frequency Filter For Biomedical Applications".

6. J. Webster, “Medical Instrumentation: Application and Design”, John Wiley, 3rd Edition, 1998.
7. Y. M. and G. M. ‘A low-noise preamplifier with adjustable gain and bandwidth for bio-potential recording applications, IEEE international symposium on circuits and systems (ISCAS) (pp. 27–30) 2007.
8. W. C. Crawford, S. C. Webb, Identifying and removing tilt noise from low-frequency (<1 Hz) seafloor vertical seismic data, Bull. Seism. Soc. Am., 90, 952-963, 2000.
9. R. D. Kosian, N. V. B. Pykhov, B. L. Edge, Coastal processes in tide less seas, ASCE Publications, 2000.
10. Yin, M. and G. M., “A low-noise preamplifier with adjustable gain and bandwidth for bio-potential recording applications”, IEEE international symposium on circuits and systems (ISCAS) (pp. 27–30) 2007.
11. W. Ahmad and R. El-Khazali, “Fractional order passive low-pass filters”, in Proceedings of the ICECS, pp. 160-163, 2003.
12. K. M. Sivrama, S. Das, K. Biswas, B. Goswami,“Fabrication of a Fractional Order Capacitor with Desired Specifications: A Study on Process Identification and Characterization,” IEEE Transactions on Electron Devices, 2011, vol 58, pp. 4067-4073.
13. K. Biswas, S. Sen, and P. Dutta, “Realization of a constant phase element and its performance s tudy in a differentiator circuit”, IEEE Transaction Circuits syst.-II, vol. 53, pp. 802-806, 2006.
14. D. Mondal and K. Biswas, “Performance study of fractional order integrator using single component fractional order element”, IET Circuits, Devices and Systems, vol. 5, pp. 334-342, 2011.
15. S. Swain, M. C. Tripathy, and S. Behera, “Particle Swarm Optimization based Bandpass Filter Using Switched-Fractional Capacitors”, in IETE Journal of Research, 2022. (Accepted).
16. S. Swain, S. Sahoo, M. C. Tripathy, and S. Behera, “Modelling of Inverse Fractional-Order Band-pass and Band-stop Filters using Fractional-Elements”, in IEEE International Conference on Distributed Computing and Electrical Circuits and Electronics (ICDCECE), 2022.
17. S. Swain, D. Mohapatra, M. C. Tripathy, and S. Behera, “Design Study of DC-DC Fractional-Order Boost-Converter using Fractional-Capacitor”, in IEEE 7th International Conference for Convergence in Technology (I2CT), 2022.
18. S. Swain, K. Biswal, M.C. Tripathy and S.K. Kar (2020), “Performance Analysis of fractional-order Sallen-Key High-pass Filter Using Fractional Capacitors”, IEEE International Conference on Computational Intelligence for Smart Power System and Sustainable Energy, Vol.1, pp.1-5.
19. K. Biswal, S. Swain, M. C. Tripathy, and S. K. Kar (2021), “Modeling and Performance Improvement of fractional-Order Band-pass Filter Using Fractional Elements”, IETE Journal of Research https://doi.org/10.1080/03772063.2021.1906334.
20. K. Biswal, S. Swain, M. C. Tripathy, and S. K. Kar (2022), “Re-configurable Band-stop and All-pass Filter Using Fractional-Order Topology”, in International Journal of Advanced Technology and Engineering Exploration (IJATEE), ACCENTS publications, 2022. Research,