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ESTIMATION AND CANCELLATION OF THE FREQUENCY OFFSET IN OFDM SYSTEM USING EXTENDED KALMAN FILTERING

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ABSTRACT

In this paper, the Extended Kalman filtering is introduced in the OFDM system in the presence of frequency offset between the transmitter and the receiver. The frequency offset degrades the performance of the OFDM system due to Inter-carrier interference (ICI).The proposed method is compared with the ICI self-cancellation (SC) scheme and OFDM system. Theperformance of OFDM system in the presence of frequency offset between the transmitter and the receiver has been analyzed in terms of the Carrier-to-Interference ratio (CIR) and Bit Error Rate.

1 INTRODUCTION

ICI self-cancellation is a scheme that was introduced by Yuping Zhao and Sven-Gustav Häggman in 2001 in [1] to combat and suppress ICI in OFDM systems.The need arises to increase bit rates in digital mobile radio communication systems [2], inter-symbol interference (ISI) and fading is threatening in conventional single carrier systems. In mobile radio situation, the relative movement between transmitter and receiver causes Doppler frequency shifts and carriers can never be perfectly synchronized. These random frequency errors in OFDM system distort orthogonality between sub-carriers and results ICI. In such systems, literature show that, the BER increases rapidly with increasing frequency offsets. Researchers have proposed various methods to combat the ICI in OFDM systems. The existing approaches that have been developed to reduce ICI can be categorized as frequency domain equalization as in [3-4], Time domain windowing as in [5] and ICI self-cancellation (SC) scheme as in [6]. Hence this has motivated to explore the performance of EKF in ICI cancellation in an OFDM system.

2. IMPULSE RESPONSE OF MOBILE RADIO CHANNELS

In mobile radio environment, the time variant impulse response model of the multipath channel is defined as

$$h(t)=\sum_{i=0}^{M-1} h_i e^{j(2\pi f_{Di}(t)+\theta_i)}\delta(t-\tau_i) \tag{1}$$

Where, M is the total number of propagation paths, $f_{Di}(t)$ is the Doppler frequency at time t of i[th] path, θ_i is the initial angle of the i[th] path and it is assumed to be 0without loosing generality, τ_i is the time delay of the i[th] path.

Normally, the changes of the $f_{Di}(t)$ are not very fast, therefore a constant value $f_{Di}(t)$ is assumed within each data block in OFDM systems. Furthermore, the frequency offset [6-10] is normalized by the sub-carrier frequency separation Δf to define a new parameter ε_i

$$\varepsilon_i = \frac{f_{D_i}}{\Delta f} \tag{2}$$

The variable ε_i is called the normalized frequency offset of i[th] path. It is a more efficient parameter when analyzing frequency offset impact in OFDM systems. Using discrete time domain index n instead of t, the channel impulse response model for one data block is expressed as:

$$h(n) = \sum_{i=0}^{M-1} h_i e^{j\frac{2\pi}{N}\varepsilon_i(n-n_i)} \tag{3}$$

Where Nis the number of sub-carriers, n_i is the delay chip number of the i[th] path. For each path, the amplitude of h_i is Rayleigh distributed.

2.1 MODELING OF FREQUENCY OFFSET

Doppler shifts can be neglected in indoor environments. Frequency offset can be caused by Doppler shift due to relative motion between the transmitter and receiver, or by differences between the frequencies of the local oscillators at the transmitter and receiver. Here the frequency offset is modeled as a multiplicative factor introduced inthe channel, as shown in Figure 1

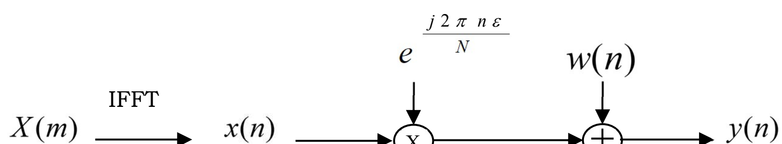


Figure 1 Frequency offset Model

The received signal is given by,

$$y(n) = x(n)e^{\frac{j2\pi n\varepsilon}{N}} + w(n) \tag{4}$$

In the above equation, normalized frequency offset $\varepsilon = \Delta f\, N T_s$. The symbol period of the sub-carrier is T_S, Δf is the frequency difference between the transmitted and received carrier frequencies and $w(n)$ is the AWGN introduced in the channel. The effect of frequency offset on the received symbol stream can be implicit by considering the received symbol $Y(k)$ on the k^{th} sub-carrier $(k = 0,1,.....N-1)$.

$$Y(k) = X(k)S(0) + \sum_{l=0, l\neq k}^{N-1} X(l)S(l-k) + n_k \tag{5}$$

where $X(k)$ is the transmitted symbols using binary phase shift keying (BPSK) for the k^{th} sub-carrier, N is the total number of sub-carriers, n_k is the noise sample of $w(n)$, and $S(l-k)$

are the complex coefficients intended for the ICI components. These are the interfering signals transmitted on sub-carriers other than the k^{th} sub-carrier. Then the intended coefficients are given by

$$S(l-k) = \frac{\sin(\pi(l+\varepsilon-k))}{N\sin(\pi(l+\varepsilon-k)/N)}\exp\left(j\pi\left(1-\frac{1}{N}\right)(l+\varepsilon-k)\right) \tag{6}$$

The quality of the signal is indicated bycarrier-to-interference ratio (CIR), which is the ratio of the signal power to the power in the interference components. It has been derived from equation 5 in [11] and is given by

$$CIR = \frac{|S(k)|^2}{\sum_{l=0,\, l\neq k}^{N-1} |S(l-k)|^2} = \frac{|S(0)|^2}{\sum_{l=0}^{N-1} |S(l)|^2} \tag{7}$$

3. EXTENDED KALMAN FILTERING TO OFDM SYSTEM

A state-space model of the discrete Kalman filter is defined as

$$z(n) = a(n)d(n) + w(n) \tag{8}$$

In this model, the observation *z(n)* has a linear relationship with the desired value *d(n)*. By using the discrete Kalman filter, *d(n)* can be recursively estimated based on the observation of *z(n)* and the updated estimation in each recursion is optimum in the minimum mean square sense[12].

$$y(n) = x(n)\, e^{j\frac{2\pi n'\varepsilon(n)}{N}} + w(n) \tag{9}$$

It is obvious that the observation y(n) is in a nonlinear relationship with the desired value $\varepsilon(n)$, i.e.

$$y(n) = f(\varepsilon(n)) + w(n) \tag{10}$$

$$\text{where} \quad f(\varepsilon(n)) = x(n)\, e^{j\frac{2\pi n'\varepsilon(n)}{N}} \tag{11}$$

In order to estimate $\varepsilon(n)$ efficiently in computation, it is built an approximate linear relationship using the first-order Taylor's expansion

$$y(n) \approx f(\hat{\varepsilon}(n-1)) + f'(\hat{\varepsilon}(n-1))[\varepsilon(n) - \hat{\varepsilon}(n-1)] + w(n) \tag{12}$$

Where $\hat{\varepsilon}(n-1)$ is the estimation of $\varepsilon(n-1)$

$$f'(\hat{\varepsilon}(n-1)) = \left.\frac{\partial f(\varepsilon(n))}{\partial \varepsilon(n)}\right|_{\varepsilon(n)=\hat{\varepsilon}(n-1)} = j\frac{2\pi n'}{N}x(n)e^{j\frac{2\pi n'\varepsilon(n-1)}{N}} \tag{13}$$

Define

$$\begin{aligned} z(n) &= y(n) - f(\hat{\varepsilon}(n-1)) \\ d(n) &= \varepsilon(n) - \hat{\varepsilon}(n-1) \end{aligned} \tag{14}$$

And the following relationship

$$z(n) = f'(\varepsilon(n-1))d(n) + w(n) \tag{15}$$

which has the same form as equation 8, i.e. *Z(n)* is linearly related to*d(n).* Hence the normalized frequency offset $\varepsilon(n)$ can be estimated in a recursive procedure similar to the discrete Kalman filter. As linear approximation is involved in the derivation, the filter is called the extended Kalman filter (EKF). The derivation of the EKF is omitted in this report for the sake of brevity. The EKF provides a trajectory of estimation for $\varepsilon(n)$. The error in each update decreases and the estimate becomes closer to the ideal value during iterations. It is noted that the actual error in each recursion between $\varepsilon(n)$ and $\hat{\varepsilon}(n)$ does not strictly obey equation 15. However it has been proved that EKF is a very useful method of obtaining good estimates of the system state. In the following estimation using the EKF, it is assumed that the channel is slowly time varying so that the time-variant channel impulse response can be approximated to be quasi-static transmission of one OFDM frame. Hence the frequency offset is considered to be constant during a frame. The preamble preceding each frame can thus be utilized as a training sequence for estimation of the frequency offset imposed on the symbols in this frame. Furthermore, in estimation, the channel is assumed to be flat-fading and ideal channel estimation is available at the receiver. Therefore in the derivation and simulation, the one-tap equalization is temporarily suppressed.

3.1 ICI CANCELLATION

There are two stages in the EKF scheme to mitigate the ICI effect, the offset estimation scheme and the offset correction scheme. The main reason for ICI is the normalized frequency offset. So first consider the estimate using EKF algorithm and then to correct using offset model.

3.2 OFFSET ESTIMATION SCHEME

To estimate the quantity $\varepsilon(n)$ using an EKF in each OFDM frame, the state equation is built as

$$\varepsilon(n) = \varepsilon(n-1) \tag{16}$$

i.e., in this case, an unknown constant ε is being estimated and that is distorted by a non-stationary process $x(n)$, an observation of which is the preamble symbols preceding the data symbols in the frame.

The observation equation is

$$y(n) = x(n)\, e^{j\frac{2\pi n'\varepsilon(n)}{N}} + w(n) \tag{17}$$

where $y(n)$ denotes the received preamble symbols distorted in the channel, $w(n)$ the AWGN, and $x(n)$ the IFFT of the preambles $x(k)$ that are transmitted, which are known at the receiver. Assume there are N_p preambles preceding the data symbols in each frame are used as a training sequence and variance σ^2 of the AWGN $w(n)$ is stationary. The computation procedure is described as follows.

1. Initialize the estimate $\hat{\varepsilon}(0)$ and corresponding state error $P(0)$.
2. Compute the $H(n)$, the derivative of y(n) with respect to $\hat{\varepsilon}(n)$ at $\hat{\varepsilon}(n-1)$. The estimate is obtained in previous iteration.
3. Compute the time-varying Kalman gain k(n) using the error variance p(n-1), $H(n)$ and σ^2
4. Compute the estimate $\hat{y}(0)$ using x (n) and $\hat{\varepsilon}(n-1)$ i.e. based on the observations up to time n-1, compute the error between the true observation y(n) and $\hat{y}(0)$
5. Update the estimate $\hat{\varepsilon}(n)$ by adding K(n) weighted error between the observation y(n) and $\hat{y}(n)$ to previous estimation $\hat{\varepsilon}(n-1)$.
6. Compute the state error P(n) with the Kalman gain K(n), H(n) and the previous error P(n-1)
7. If n < N_p increment n by 1 and go to step 4; otherwise stop.

It is observed that the actual errors of the estimation $\hat{\varepsilon}(n)$ from the ideal value $\varepsilon(n)$ are computed in each step and are used for adjustment of estimation in next step. Through the recursive iteration procedure described above, an estimate of the frequency offset ε can be obtained.

The pseudo code (for EKF) of computation is summarized as follows:

Initialize state error P(n), estimate $\hat{\varepsilon}(0)$

For $n=1,2,.....,N_p$ Compute Kalman gain

$$H(n)=\frac{\partial y(x)}{\partial x}\Big/ x=\hat{\varepsilon}(n) = \frac{j2\Pi n'}{N} e^{\frac{j2\Pi n' \hat{\varepsilon}(n-1)}{N}} x(n) \tag{18}$$

$$K(n)=p(n-1)H^*(n)\left[p(n-1)+\sigma^2\right]^{-1} \tag{19}$$

Update the estimate

$$\hat{\varepsilon}(n)=\hat{\varepsilon}(n-1)+\mathrm{Re}\left\{K(n)\left[y(n)-x(n)e^{\frac{j2\Pi n' \hat{\varepsilon}(n-1)}{N}}\right]\right\} \tag{20}$$

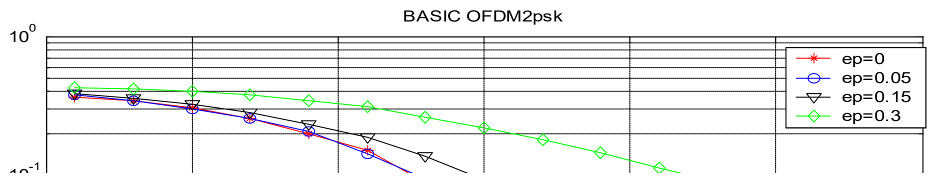
State error

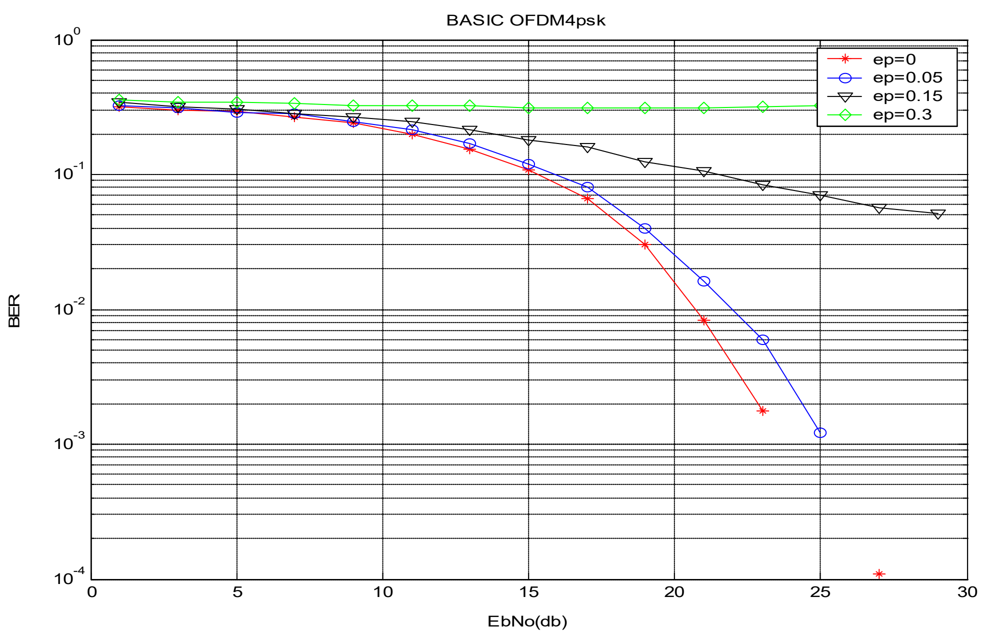
$$P(n)=\left[1-K(n)H(n)\right]p(n-1) \tag{21}$$

4. SIMULATION RESULTS

In order to compare the two different cancellation schemes, BER curves were used to evaluate the performance of each scheme. For the simulations MATLAB was employed with its Communications Toolbox for all data runs. Frequency offset was introduced as the phase rotation as given by equation (4). Modulation schemes of binary phase shift keying (BPSK), 4-PSK, Quadrature amplitude modulation (QAM) was chosen and simulations for cases are taken for normalized frequencyoffsets equal to 0, 0.05, 0.15, and 0.30.

Figure 2 BER performance of a standard OFDM system





without ICI cancellation for 2- PSK modulation

Figure 3 BER performance of a standard OFDM system without ICI cancellation for 4-PSK modulation

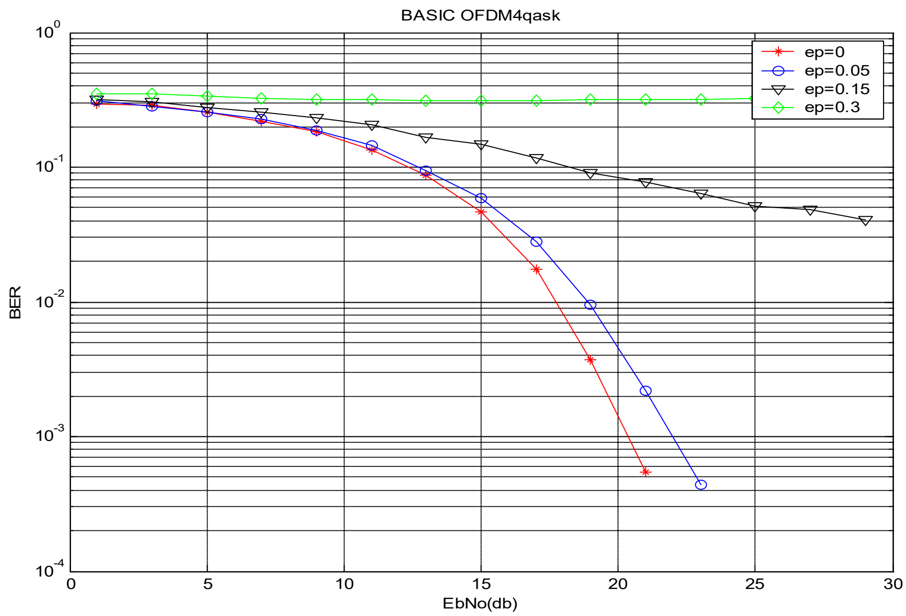


Figure 4 BER performance of a standard OFDM system without ICI cancellation for QASK modulation

Here 52 sub-carriers containing data and an IFFT size of 128 is used. Actual bits transmitted are 5200 for M=2 modulation and 10400 for M=4 modulation. After encoding the bits size becomes 9100 for M=2 and 18200 for M=4 modulation respectively. After IFFT the total number of OFDM symbols becomes 175 for both modulations. These results show that degradation of performance increases with frequency offset. For the case of BPSK, even severe frequency offset of 0.30 does not deteriorate the performance too greatly.

However, for QAM with an alphabet of size 2, performance degrades more quickly. When frequency offset is small, the 4-QAM system has a lower BER than the BPSK system. But the BER of 4-QAM varies more dramatically with the increase in frequency offset than that of BPSK. Therefore it is concluded that larger alphabet sizes are more sensitive to ICI. From the results it can be compared in analytic manner for E_BN_O=11, normalized offset=0.15. For BPSK modulation in 5200 transmitted bits 947 errors occur with BER=0.1821. For 4-PSK modulation in 10400 transmitted bits 2466 errors occur with BER=0.2371.For 2-QAM modulation in 5200 transmitted bits 948 errors occur with BER=0.1843. For 4-QAM modulation in 10400 transmitted bits 2061 errors occurred with BER=0.1982. So it is able to observe that as the alphabet size increases ICI also increases. It can also be observed that for M=2 numbers of errors are almost same for PSK and QAM. But for M=4 number of errors are less for QAM when compared to PSK.

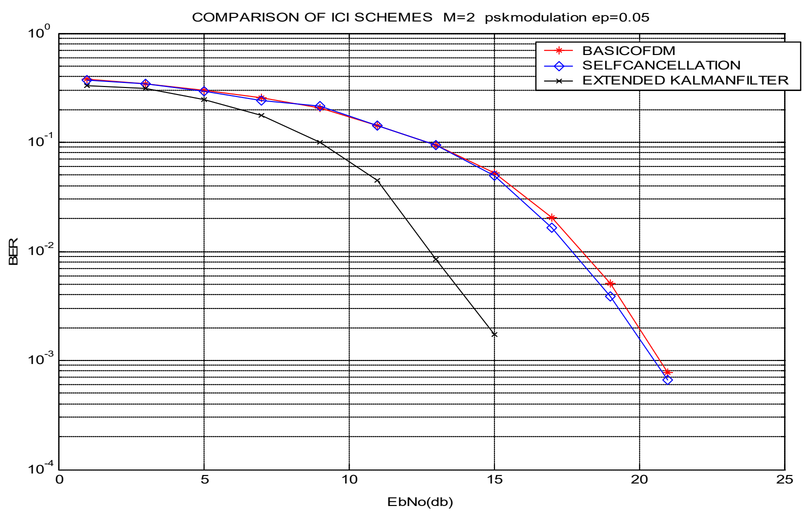


Figure 5 BER Performance M=2 (PSK) with ICI Cancellation, ε=0.050

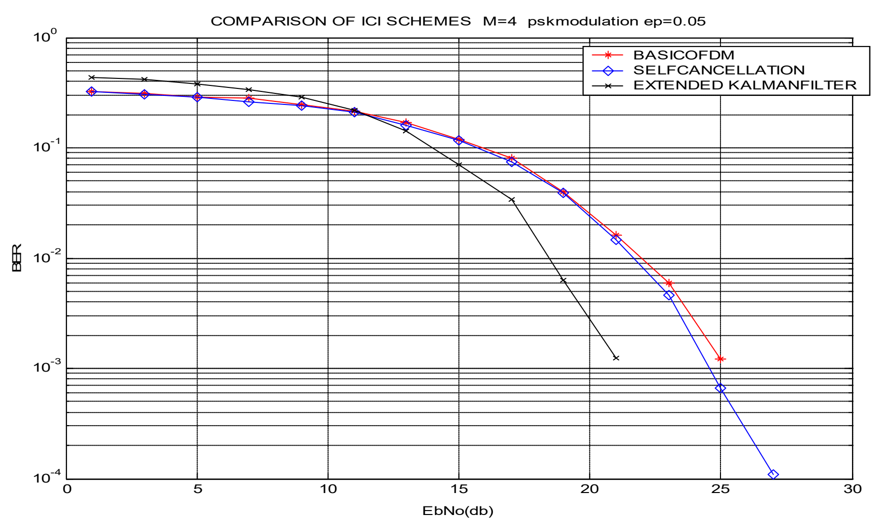


Figure 6 BER Performance M=4 (PSK) with ICI Cancellation, ε=0.050

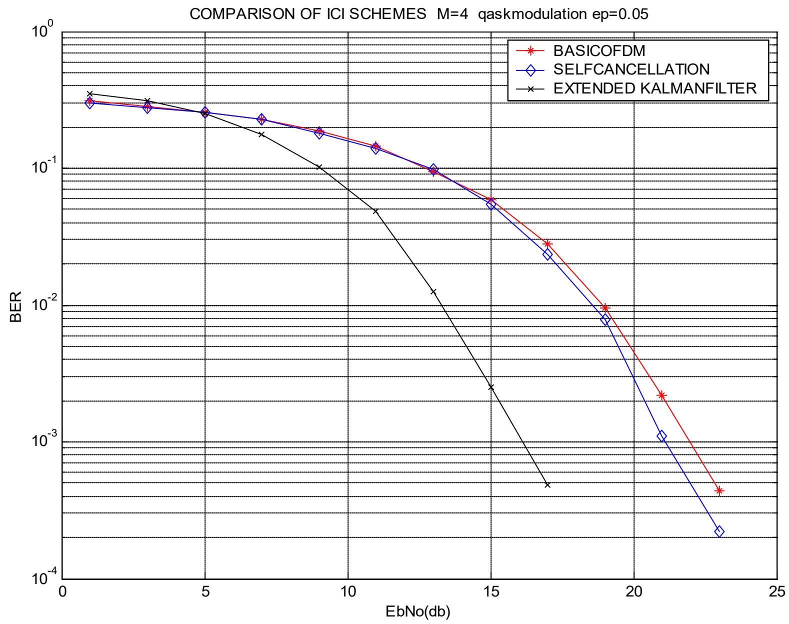


Figure 7 BER Performance M=4 (QASK) with ICI Cancellation, ε=0.050

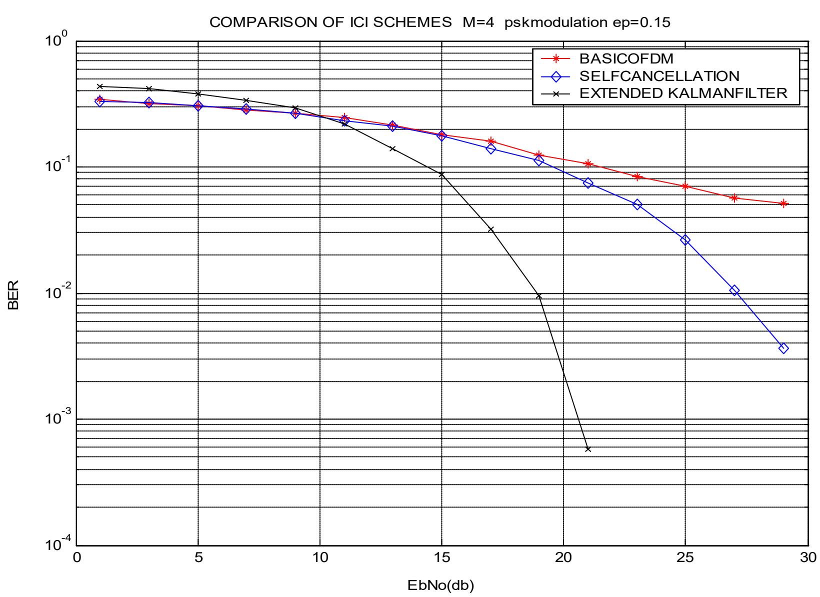


Figure 8 BER Performance M=4 (PSK) with ICI cancellation ε=0.150

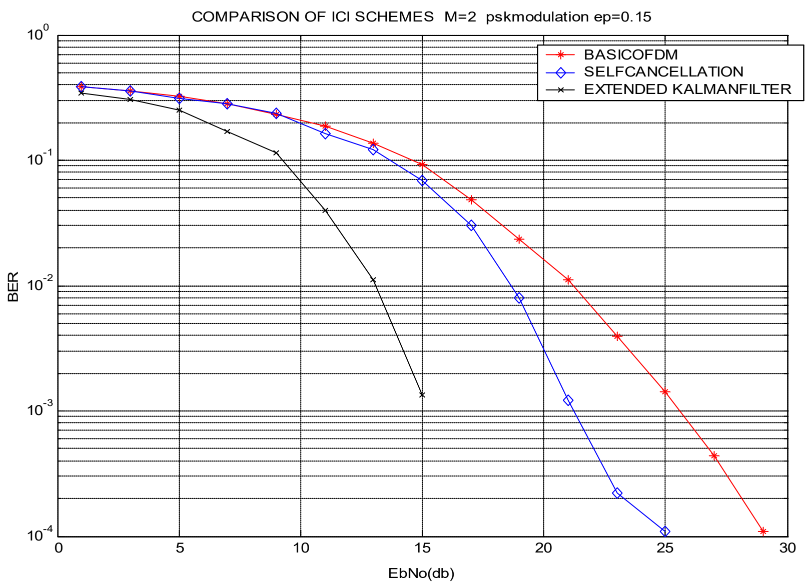


Figure 9 BER Performance M=2 (PSK) with ICI cancellation ε=0.150

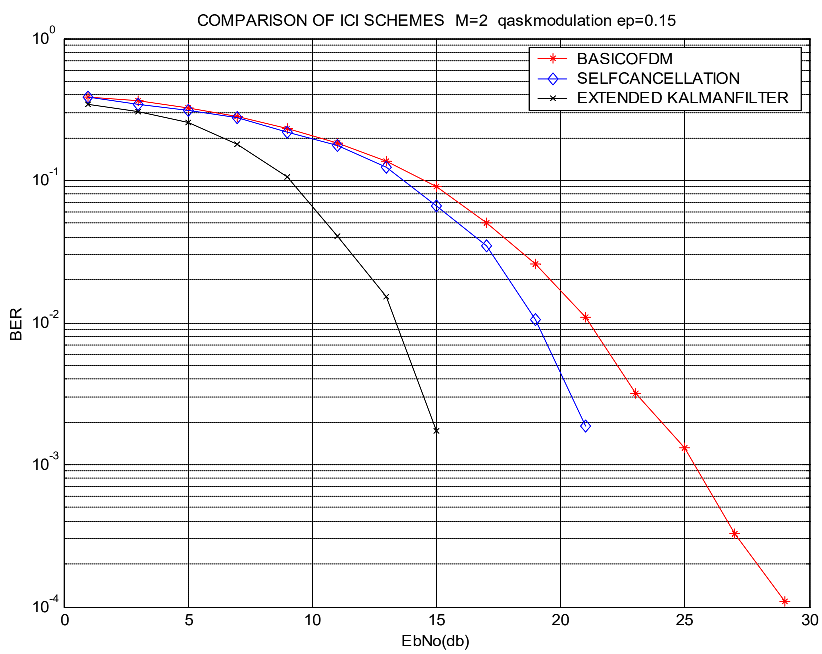


**Figure
10 BER Performance M=2 (QASK) with ICI cancellation
ε=0.150**

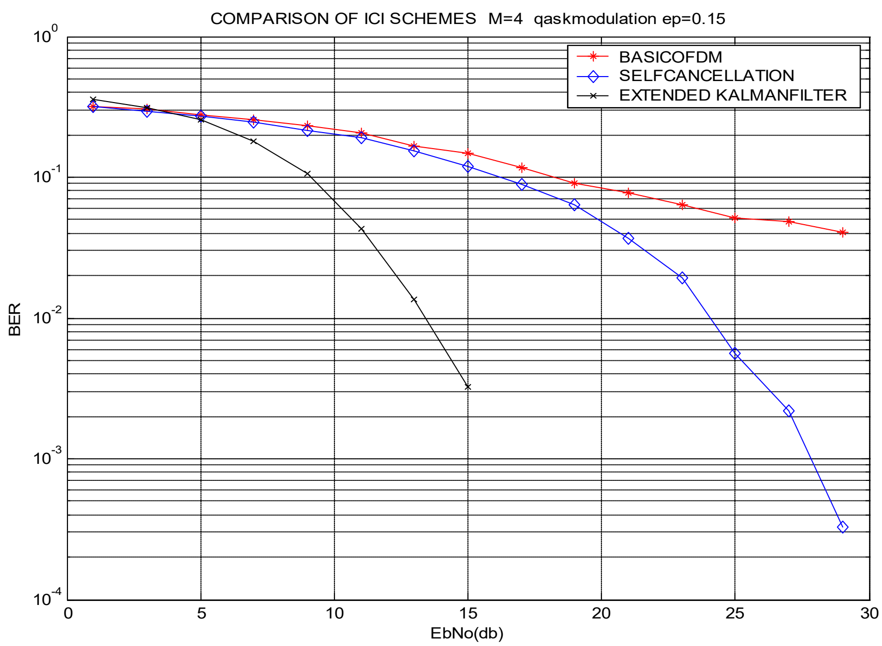


**Figure 11 BER Performance M=4 (QASK) with ICI cancellation
ε=0.150**

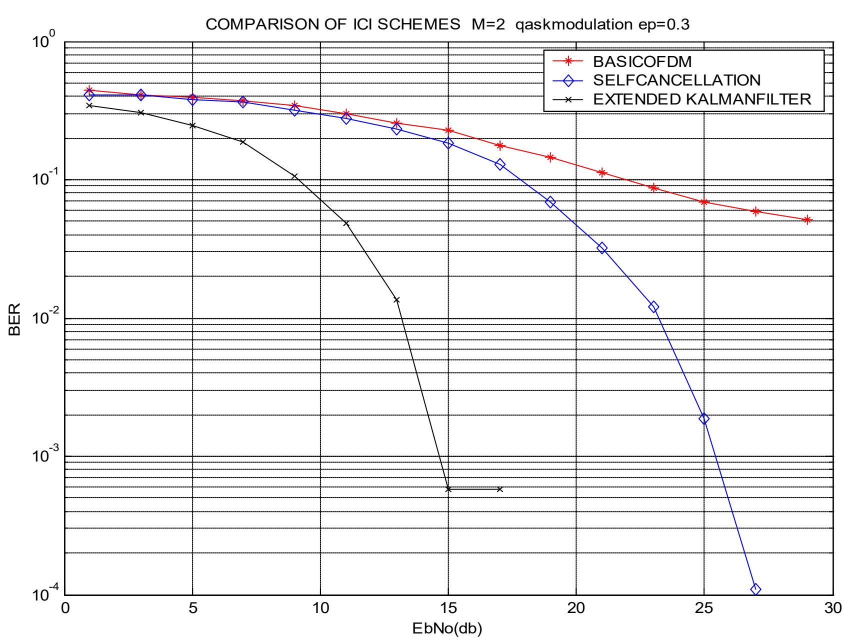


Figure 12 BER Performance M=2 (QASK) with ICI Cancellation ε=0.3

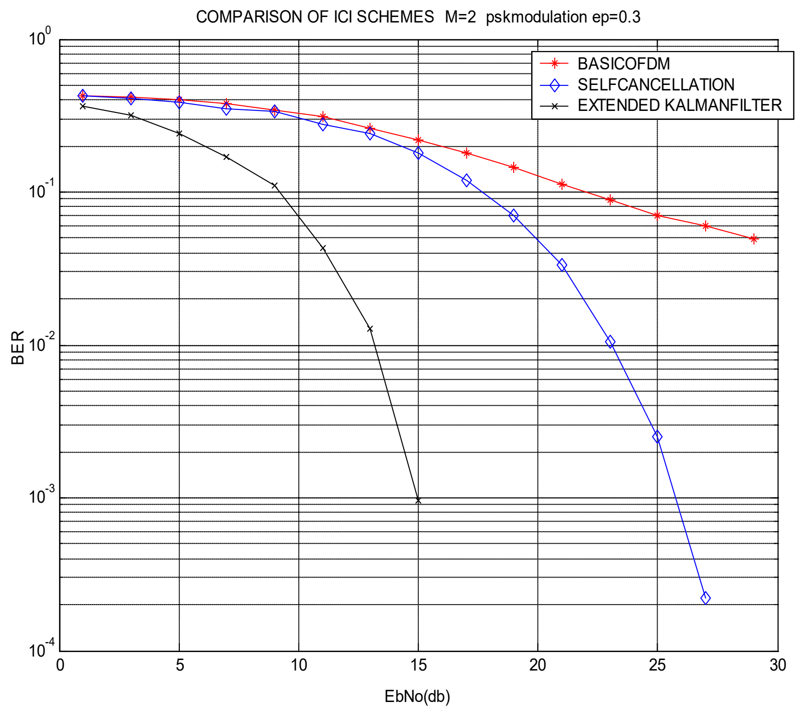


Figure 13 BER Performance M=2 (PSK) with ICI Cancellation, ε=0.30

Figures 8 to 13 provide comparisons of the performance of the SC and EKF schemes for different alphabet sizes and different values of the frequency offset. It is observed in the figures that each method has its own advantages. In the presence of small frequency offset and binary alphabet size, self-cancellation gives the best results [13]. However, for larger alphabet sizes and larger frequency offset such as 4-QAM and frequency offset of 0.30, self-cancellation does not offer much increase in performance. The Kalman filter method indicates that for very small frequency offset, it does not perform very well, as it hardly improves BER. However, for high frequency offset the Kalman filter does perform extremely well. It gives a significant boost to performance.

For M=2 PSK modulation in 5200 transmitted bits, 1135 errors occur with a BER of 0.047473 for offset of 0.3 and E_BN_O = 15 for Basic OFDM. In the similar conditioned using self-cancellation scheme 905 errors occur with a BER of 0.17407. If EKF estimation is used then only 4 errors occur with BER of 0.00076943.For M=4 QASK modulation in 10400 transmitted 2042 bits errors occurred with a BER of 0.19637 for offset of 0.05 and E_BN_O = 09 dB for Basic OFDM. In the similar conditioned using self-cancellation scheme 1885 errors occurred with a BER of 0.18121. If EKF estimation is used then 1094 errors occurred with BER of 0.10519. Tables 1-4 summarize required values of SNR for BER specified at 10^{-2}. Significant gains in performance can be achieved using EKF methods for a large frequency offset.

Table 1 Required SNR and improvement for BER of 10^{-2} for 2-PSK

Method	ε=0	Gain	ε=0.05	Gain	ε=0.15	Gain	ε=0.3	Gain
Basic OFDM	17.8dB		18dB		21dB		31dB	
SC	17.6dB	0.2dB	17.6dB	0.4dB	18.8dB	2.2dB	23dB	8dB
EKF	13 dB	4.8dB	13dB	5 dB	12.9dB	8.1dB	13dB	18dB

Table 2 Required SNR and improvement for BER of 10^{-2} for 4-PSK

Method	ε =0	Gain	ε=0.05	Gain	ε=0.15	Gain	ε=0.3	Gain
Basic OFDM	20.8dB		21.8dB		31dB		35dB	
SC	20.5dB	0.3dB	21.6dB	0.2dB	27dB	4dB	31dB	4dB
EKF	19dB	1.8dB	18.5dB	3.3dB	19dB	14dB	19dB	16dB

Table 3 Required SNR and improvement for BER of 10^{-2} for 2-QAM

Method	ε=0	Gain	ε=0.05	Gain	ε=0.15	Gain	ε=0.3	Gain
Basic OFDM	17.8dB		18dB		21dB		35dB	
SC	17.8dB	0dB	18dB	0dB	19dB	2dB	23dB	11dB
EKF	13dB	4.8dB	13dB	5dB	13dB	8dB	13dB	22dB

Table 4 Required SNR and improvement for BER of 10^{-2} for 4-QAM

Method	E =0	Gain	ε=0.05	Gain	ε=0.15	Gain	ε=0.3	Gain
Basic OFDM	18dB		19dB		30dB		35dB	
SC	17.6dB	0.4dB	18.8dB	0.2dB	24dB	6dB	31dB	4dB

EKF	13dB	5dB	13.3dB	5.7dB	13.5dB	16.5dB	13dB	22dB

5. CONCLUSIONS

In this paper, the performance of OFDM systems in the presence of frequency offset between the transmitter and the receiver has been analyzed in terms of the Carrier-to-Interference ratio (CIR) and the BER performance. Inter-carrier interference (ICI) which results from the frequency offset degrades the performance of the OFDM system. The extended Kalman filtering (EKF) method for estimation and cancellation of the frequency offset has been investigated and compared with basic OFDM and self-cancellation.For small alphabet sizes (BPSK) and for low frequency offset values the SC and EKF techniques have good performance in terms of BER. However, for higher order modulation schemes, the EKF technique performs better. This is attributed to the fact that the EKF methods estimate the frequency offset very accurately and cancel the offset using this estimated value.

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